The Product Cycle and Inequality

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Abstract

This paper models the product cycle and explains how it relates to world inequality. In the model, both phenomena arise because skilled people have a comparative advantage in making high-tech products. The model can explain up to a 10:1 income differential between people and up to a 7:1 differential between countries. Tariff policies and intellectual-property protection have a much larger effect here than in some other models.

1 Introduction

The "Product Cycle" is the term Vernon (1968) used to describe the tendency for new products to be made in rich countries, and old products to be made in poor countries. He said this was because firms in rich places sell to the world's richest and most demanding consumer, and because in rich places labor is the most expensive and capital-intensive technology is more profitable there.

I argue that the product cycle arises instead because technologies are product specific. The world economy demands many products, and so many technologies must coexist. New products are more high tech and demand more skills to make them. The people using the best technologies will then want to raise their skills relative to those of other people. Thus the product cycle and inequality both have their origins in the complementarity between technology and skill. The main results are:

1. The calibrated version implies a 10:1 per-capita income ratio of leader and laggard. This contrasts to Lucas (1988), e.g., where any income distribution is an equilibrium.

2. World inequality depends exclusively on the efficiency gap between successive technologies. The technology frontier can grow in many little steps, or in a few large ones. The latter case that produces more inequality, because the technologies in use will then be more dispersed.

3. Reducing world-wide patent protection from 18 to 6 years, e.g., impoverishes the world by a factor of 2. A 20% tariff on the import of technology reduces a country's output by a factor of 5.

The effects are large partly because the model assumes away some important frictions in the market for technology. It assumes away all costs of switching technologies such as costs of learning a new technology and costs of reallocating technology-specific assets, and it takes the protection of intellectual property to be perfect. In return, results are all derived by hand and clearly.

2   Model

A world market exists for final goods, for intermediate goods, for the research input, and for research output, i.e., technologies.

Final-goods.—Final goods producers are competitive. The production function is

$$y = \left(\int_0^\infty x_i^\alpha di \right)^{1/\alpha},$$

where x_i is the i'th intermediate good. Let P_i be the price of good i in units of the final good. The final-goods producers problem is

$$\max_{(x_i)_0^\infty} \left\{ y - \int_0^\infty P_i x_i di \right\}$$

with the first-order condition

$$y^{1-\alpha} x_i^{\alpha-1} - P_i = 0. \tag{1}$$

Demand is elastic and total revenue, $P_i x_i = y^{1-\alpha} x_i^\alpha$, always rises with output.

Intermediate goods.—With his skill, s, an intermediate-goods producer can make

$$x = z s^\beta \tag{2}$$

units of good z. From now on we shall refer to a good by its efficiency, z. Let $p(z)$ be the period license fee for making good z. This yields a profit of

$$Px - p(z) = y^{1-\alpha} z^\alpha s^{\beta\alpha} - p(z),$$

The objective of the intermediate-goods producers is to maximize this quantity by selecting the technology, z, to license.

The supply of inventions.—A product's z is constant over its lifetime. New products, with higher z's are invented at a constant rate, N, to be determined later. Each is retired at age T which, for now, is also given. The age distribution of goods is then uniform on the interval $[0, T]$. Assume that

$$z_{\max}(t) = e^{gt}.$$

For now, g too is given. We need first the stationary distribution of product quality conditional on g, and conditional on T. This distribution shifts over time but it always has the same shape. We shall describe its state at $t = 0$. Let τ denote a technology's age at $t = 0$. Then that technology's quality is $z_\tau = e^{-g\tau}$. Then the worst technology in use is of quality e^{-gT}. Each agent makes a different good. Therefore, the number of goods equals the number of agents which we normalize to 1. That is, since the product's quality, z, relates to its age τ, via $\ln z = -g\tau$, we have

Lemma 1 *If τ is uniform on $[0, T]$, then $\ln z$ is uniform on $[-gT, 0]$ with density $1/gT$. The density of z is*

$$m(z) = \left(\frac{1}{gT}\right)\frac{1}{z}, \quad \text{for } z \in [e^{-gT}, 1]. \tag{3}$$

Then $\ln z_t$ is uniform on $[g(t - T), gt]$, and $m_t(z) = \left(\frac{1}{gT}\right)\frac{1}{z}$ for $z \in [e^{g(t-T)}, e^{gt}]$. This all hinges on an exogenous arrival of new products at a uniform rate and the growth of frontier efficiency at the rate g.

2.1 The market for licenses

In contrast to Krugman (1979) all agents can make any product, and in contrast to Eaton and Kortum (1999) technology diffusion is endogenous. It is determined in the market for licenses. To make product z at a given date, a firm must pay its per-period license fee $p(z)$. Let us assume only one producer per product, derive the prices at which all markets clear, and then verify that no one has the incentive to enter a market as a second producer.

We start, then, with a one-to-one assignment with side payments – the "transferable-utility" case. Taking the distributions of z and s as given, let us find the market-clearing license-fee function $p_t(z)$.

The technology-adoption decision: We shall assume that[1]

$$\alpha = \frac{1}{1 + \beta}. \tag{4}$$

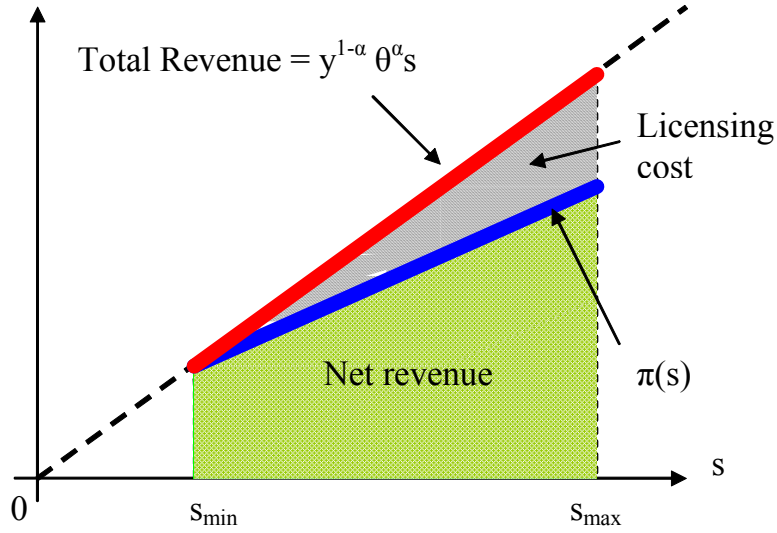[1] This assumption gets extensive scrutiny in Section 5.

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Figure 1: THE BREAKDOWN OF INCOME INTO LICENSING FEES AND PROFITS

Taking his skill-level s as given, a monopolist then solves

$$\pi(s) = \max_{z}\{y^{1-\alpha}z^{\alpha}s^{1-\alpha} - p(z)\}.$$

Thus (4) induces constant returns in (z, s). Revenue increases with output and the firm always produces at full capacity. The first-order condition reads

$$\alpha\left(\frac{sy}{z}\right)^{1-\alpha} - p'(z) = 0.$$

Evidently, then, for any $\theta > 0$ which, for now, is given, the assignment

$$z = \theta s \tag{5}$$

is an equilibrium if

$$p(z) = \gamma(z - z_{\min}), \tag{6}$$

where

$$\gamma = \alpha\theta^{\alpha-1}y^{1-\alpha},$$

and if the appropriate market-clearing conditions, and "corner" conditions hold. The corner condition concerns the worst product, $z_{\min}$: Since old products are dropped, $p(z) = 0$ for $z < z_{\min}$. By continuity, $p(z_{\min}) = 0$.

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Technology-market clearing.—Let $n(s)$ be the date-zero density of s. License-market clearing at $t = 0$ requires that for all $z \in \left[e^{-gT}, 1\right]$,

$$\int_z^1 m(v)dv = \int_{z/\theta}^{1/\theta} n(s)ds. \tag{7}$$

Proposition 1 *For any positive (g, T, θ), (6) and (5) constitute an assignment equilibrium when the distributions z and s are given by (3) and (9), in which market clearing (7) also holds.*

Note some properties of this equilibrium. First, $\pi(s)$ is linear in s:

$$\pi(s) = y^{1-\alpha}\theta^{\alpha}\left(\alpha s_{\min} + [1 - \alpha]s\right). \tag{8}$$

Second, output, $y^{1-\alpha}\theta^{\alpha}s$, and license fees, $p(\theta s) = \gamma\theta(s - s_{\min}) = \alpha\theta^{\alpha}y^{1-\alpha}(s - s_{\min})$ are linear in s. Figure 1 illustrates the situation.

Now, according to (5), it must be that for all $z \in \left[-\frac{T}{g}, 0\right]$, $\ln s = \ln z - \ln\theta$, which implies the claim:

Proposition 2 *If τ is uniformly distributed on $[0, T]$, $\ln s$ is uniformly distributed on $[-gT - \ln\theta, -\ln\theta]$ with density $1/gT$.*

The distribution of s itself is

$$n(s) = \left(\frac{1}{gT}\right)\frac{1}{s}, \quad \text{for } s \in [\frac{1}{\theta}e^{-gT}, \frac{1}{\theta}]. \tag{9}$$

Thus $s_{\max} = \theta$ and $s_{\min} = \theta e^{-gT}$, as illustrated in Figure 2.

The no-switching condition.—We assumed monopoly in each product. No firm should want to enter as a second firm in someone else's market. Under Bertrand competition, production will have to be at full capacity of all the firms in that market.[2] Suppose firm s_0 invades firm s's market. It can do so only if it pays the license fee $p(\theta s)$. Industry output would then be $z\left(s^\beta + s_0^\beta\right) = \theta s\left(s^\beta + s_0^\beta\right)$, and from (1). Its payoff from doing so must be less than its payoff in its own market:

$$y^{1-\alpha}\left(\theta s\left[s^\beta + s_0^\beta\right]\right)^{\alpha - 1}zs_0^\beta - p(s) \leq \pi(s_0). \tag{10}$$

The Appendix proves

Proposition 3 *(10) holds for all (s, s_0) between $\frac{1}{\theta}e^{g(t-T)}$ and $\frac{1}{\theta}e^{gt}$.*

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[2] I assume that all of the firms involved must stick to their equilibrium values of u_I and u_R. If this is relaxed, the analysis acquires many of the intricacies of incumbent-challenger analyses of natural monopoly.

$$n_t(s) = \left(\frac{1}{gT}\right)\frac{1}{s}$$

Skill distribution at $t = 0$

Skill distribution at $t > 0$

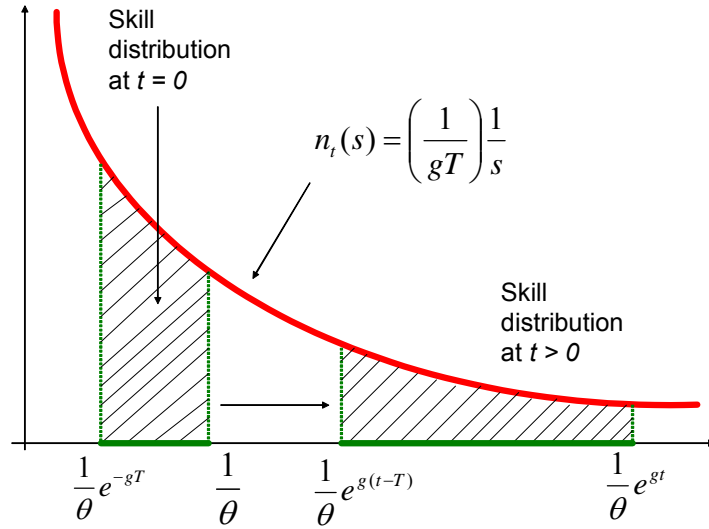$\frac{1}{\theta}e^{-gT}$　$\frac{1}{\theta}$　$\frac{1}{\theta}e^{g(t-T)}$　$\frac{1}{\theta}e^{gt}$

Figure 2: THE MOVEMENT OF $n_t(s)$ OVER TIME

This establishes that the one-to-one assignment is indeed an equilibrium. All this is conditional on $\theta, g,$ and T which will be determined later.

Re-shuffling of assignments.—A product has a constant z, but the skill of each infinitely-lived agent will grow. Thus the assignment $z = \theta s$ can hold at each t only if products move down the skill distribution. When new, a product is assigned to $s_{\max}$. By the time it reaches age T, it is matched with $s_{\min}$.

2.2 Accumulation of skill

Intermediate-goods manufacturers own their human capital and decide how to accumulate it over time. Each has a unit of time that he divides between production (u_P), research (u_R), and human-capital investment (u_I):

$$u_P + u_R + u_I = 1. \tag{11}$$

An agent's skill supply is

$$s = u_P h.$$

Human capital investment uses only time, as in Lucas (1988):

$$\dot{h} = \eta u_I h. \tag{12}$$

Wealth maximization: We shall now solve the accumulation problem of someone who is forced to set $u_{R,t} = 0$ for all t. The solution will be the same as for people

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who can set $u_{R,t} > 0$ because the research wage per unit of of h will be the same as the return of h in production. Let $u_t \equiv u_{P,t} + u_{R,t}$. The expression in (8) pertains to period zero, but $s_{\min}$ grow at the rate g. An agent that at date t supplies skill $s_t = u_t h_t$ will receive an income

$$\pi_t(u_t h_t) = y_t^{1-\alpha} \theta^\alpha \left(\alpha e^{gt} s_{\min} + (1-\alpha) u_t h_t \right).$$

The agent solves $\max \left\{ \int_0^\infty e^{-rt} \pi_t(u h_t) dt \right\}$, but he cannot influence the term $y_t^{1-\alpha} \theta^\alpha \alpha e^{gt} s_{\min}$. Since y grows at the rate g/α, he chooses u_t to maximize $(1-\alpha) \theta^\alpha y_0^{1-\alpha} \int_0^\infty e^{-\left(r - \left(\alpha^{-1} - 1 \right) g \right) t} u_t h_t dt$, which is equivalent to the problem

$$\max_{(u_t, h_t)_0^\infty} \int_0^\infty e^{-\left(r - \left(\alpha^{-1} - 1 \right) g \right) t} u_t h_t dt, \text{ s.t. } \dot{h}_t = \eta(1 - u_t) h_t,$$

with h_0 given. The Hamiltonian is

$$\mathbf{H} = e^{-\left(r - \left(\alpha^{-1} - 1 \right) g \right) t} u h + \bar{\mu} \eta (1 - u) h,$$

Let $\mu = e^{-\left(r - \left(\alpha^{-1} - 1 \right) g \right) t} \bar{\mu}$ be the current value multiplier so that the current-value Hamiltonian is just $u h + \mu \eta (1 - u) h$. We shall only analyze constant-growth paths. Evaluated at a point at which $\dot{\mu} = 0$, the FOC's are

$$1 - \mu \eta = 0,$$

and

$$\mu \eta (1 - u) + u = (r - (1 - \alpha) g) \mu.$$

Since h drops out from these two conditions, the solution for u will not depend on h. Eliminating μ we have

$$r = \eta + \left(\alpha^{-1} - 1 \right) g. \tag{13}$$

This is an arbitrage condition equating the rate of interest to the rate of return to investing in h.

Saving.—Utility is homothetic, and we need only the world per capita consumption Given his wealth, the agent maximizes his lifetime utility:

$$\int_0^\infty e^{-\rho t} \frac{c_t^{1-\sigma}}{1 - \sigma} dt.$$

If c is to grow at the rate g, we must have:

$$g = \frac{r - \rho}{\sigma}.$$

Together with (13) this implies that

$$g = \frac{\eta - \rho}{\sigma - (\alpha^{-1} - 1)}. \tag{14}$$

2.3 Research

The research sector is competitive. .

Number of new products.—Letting H be total human capital devoted to research, the flow of new products is

$$N_t = \left[\frac{\lambda}{z_{\max}(t)} \right] H.$$

where $H_t = u_R \int_{h_{\min}(t)}^{h_{\max}(t)} hm(h)\, dh$. Good ideas are harder to find, and we assume that their number is inversely proportional to $z_{\max}$. Since $h = s/u_P$, since $z_{\max}(0) = 1$, and since $s = z/\theta$, $H_0 = \int_{-T}^{0} \left(\frac{u_R}{\theta u_P} \right) e^{g\tau} d\tau$, i.e.;

$$\frac{H_t}{z_{\max}(t)} = \left(\frac{u_R}{g\theta u_P} \right) \left(1 - e^{-gT} \right).$$

Research wage.—The supply of human capital to research is infinitely elastic because its opportunity cost is the same for all agents; by (8) it is equal to

$$w_t = (1 - \alpha)\, \theta^\alpha y_t^{1-\alpha}. \tag{15}$$

Thus a worker of quality h receives income $w_t u_R h$ from research, and $w_t u_P h$ from production.

Free-entry condition.—The value of an invention is the discounted flow of license fees. The period-t license fee of a quality-1 technology is, using (6),

$$p_t(1) = \gamma_t \left(1 - e^{gt} z_{\min} \right),$$

where $\gamma_t = \alpha \theta^{\alpha-1} y_t^{1-\alpha}$. The lifetime value of the right to license a frontier technology is $V(1) = \int_0^T e^{-rt} p_t(1)\, dt$. The free-entry condition, stated at date zero, then is

$$w = \left(\frac{\lambda}{z_{\max}} \right) V(1).$$

Since $[1 + (\alpha^{-1} - 1)]\, g = \alpha^{-1} g$,

$$
\begin{aligned}
V(1) &= \gamma_0 \left[\int_0^T e^{-\left(r - (\alpha^{-1} - 1)g \right)t}\, dt - z_{\min} \int_0^T e^{-(r - \alpha^{-1} g)t}\, dt \right] \\
&= \gamma_0 \left(\frac{1 - e^{-\eta T}}{\eta} - \frac{1 - e^{-(\eta - g)T}}{\eta - g} z_{\min} \right),
\end{aligned}
$$

because $r - (\alpha^{-1} - 1)\, g = \eta$. Since $\gamma_0 = \alpha \theta^{\alpha-1} y_0^{1-\alpha}$ and since $z_{\min} = e^{-gT}$, the free-entry condition reduces to

$$\frac{(1 - \alpha)}{\alpha} \theta = \lambda \left(\frac{1 - e^{-\eta T}}{\eta} - \frac{1 - e^{-(\eta - g)T}}{\eta - g} e^{-gT} \right). \tag{16}$$

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Quality of ideas.—Let Δ be the growth in frontier quality per new idea:

$$\frac{1}{z_{\max}} \frac{dz_{\max}}{dt} = \Delta N_t.$$

Since $z_{\max}(0) = 1$, since $N_t = N$, and since z must grow at the same rate as h, we have $z_{\max}(t) = e^{gt}$, where

$$g = \Delta N, \tag{17}$$

and where

$$N = \frac{\lambda}{\theta} \left(\frac{u_R}{u_P}\right) \frac{\left(1 - e^{-gT}\right)}{g}. \tag{18}$$

Turnover of products.—Products turn over in exactly T periods. Since population size is 1, the number of technologies invented over T periods must also add up to 1:

$$TN = 1. \tag{19}$$

Stationary equilibrium.—It consists of 6 real numbers g, T, θ, u_P, u_R, and u_I, that solve (11), (12), (14), (16), (17), and (19).

3   Properties of the model

Output.—Let us refer to a good by its efficiency z. Then using (4), the output of good z is

$$x = \theta^{-\beta} z^{1+\beta} = \theta^{-(1-\alpha)/\alpha} z^{1/\alpha}.$$

Thus if θ is a constant, the output, x, of each good z is constant over its lifetime. We then have

Proposition 4 *The world output of final goods is*

$$y_t = A e^{\frac{1}{\alpha} gt}, \tag{20}$$

where $A = \left(\frac{1}{\theta^{1-\alpha} gT} \left(1 - e^{gT}\right)\right)^{1/\alpha}$.

Proof. Each x is constant; only the window $[t - T, t]$ advances. The density of product ages is $1/T$. Then $y_t = \left(\int_{t-T}^{t} \left(e^{gt} \left[\frac{e^{gt}}{\theta}\right]^{\beta}\right)^{\alpha} \frac{1}{T} dt\right)^{1/\alpha} = \left(\frac{1}{T\theta^{\alpha\beta}} \int_{t-T}^{t} e^{gt} dt\right)^{1/\alpha}$, because $\alpha(1+\beta) = 1$. Then $y_t = \left(\frac{1}{\theta^{\alpha\beta} gT} e^{gt} \left[1 - e^{-gT}\right]\right)^{1/\alpha}$. Using (4) whence $\beta = (1-\alpha)/\alpha$, (20) follows. ∎

Growth and factor shares.—By (20), y grows at the rate g/α with g given in (14) as do the combined incomes from production of the x_i's. By (15), w grows at $(1-\alpha)\, g/\alpha$, and research incomes grow at $g+(1-\alpha)\, g/\alpha = g/\alpha$. The (date-zero) income share of research is (since $\pi(s) = y^{1-\alpha}\theta^\alpha\, (\alpha s_{\min} + [1-\alpha]\, s)$ and since the date-zero mean of s is $\bar{s} \equiv \frac{1-e^{-gT}}{\theta gT}$, the world share of income going to R&D is

$$\frac{wu_R H}{\int \pi_t(s) m(s)\, ds} = \frac{u_R}{u_P \left(1 + \frac{\alpha}{1-\alpha}\frac{s_{\min}}{\bar{s}}\right)} = \frac{u_R}{u_P \left(1 + \frac{\alpha}{1-\alpha}\frac{gTe^{-gT}}{1-e^{-gT}}\right)}.$$

Creative destruction.—Products are phased out as in Stokey (1991) and the product window marches to the right. Combining (17) with (19) gives a reduced-form relation between two endogenous variables g and T,

$$g = \frac{\Delta}{T}, \tag{21}$$

which emphasizes the creative-destruction aspect of the model: Higher growth demands faster replacement of products.

Pattern of trade.—The rich export new (intermediate) products and import old (intermediate) products as in Krugman (1979).

Inequality.—By (1), income differentials between the richest and poorest agent are

$$\frac{Y_{\max}}{Y_{\min}} = \frac{(Px)_{\max}}{(Px)_{\min}} = \left(\frac{s_{\max}}{s_{\min}}\right)^{(1+\beta)\alpha} = \frac{h_{\max}}{h_{\min}} = e^{gT}. \tag{22}$$

Moreover, the log of relative incomes should be uniformly distributed on $[-gT, 0]$ with density $1/gT$. Thus the world distribution of logged per-capita income should be uniform and should march forward at the rate g. From (21) we have this paper's main result:

Proposition 5 *Inequality depends only on Δ;*

$$\frac{Y_{\max}}{Y_{\min}} = e^\Delta.$$

That Δ alone should determine inequality is because Δ alone governs the dispersion in technological quality among the measure 1 of latest vintage technologies in use at each date. The parameter λ governs only the turnover of technologies and it has no bearing on inequality. It has a level effect on output, however: Since λ and θ enter (16) and (18) as a ratio, and since they are absent from the other equations,

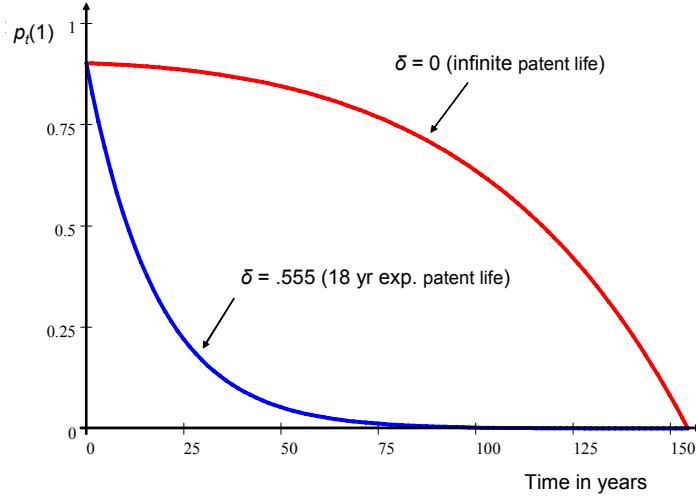**Proposition 6** *θ is proportional to λ*

Figure 3: OBSOLESCENCE OF PATENTS – $p_t(1)$

Markups.—The markup over marginal cost is

$$\frac{\pi(s)}{[w + q'(s)]s} = 1 - \alpha + \frac{s_{\min}}{s},$$

and at the baseline values of the parameters (see Table 1) it ranges from 0.19 for the highest-skilled producer to 1.09 for the lowest skilled – a little on the high side.

Obsolescence of patents.—The flow value to a patent is $p_t(z)$ in (6). For a patent issued at date zero, $z = 1$ and $z_{\min} = e^{-gT}$, and $p_t(1) = \alpha\theta^{\alpha-1}y_0^{1-\alpha}e^{(\alpha^{-1}-1)gt}\left(1 - e^{g(t-T)}\right)$. Setting $\alpha\theta^{\alpha-1}y_0^{1-\alpha} = 1$, Figure 3 plots $p_t(1)$ at the benchmark parameter values as t ranges from zero to $T = 154.5$. This is the top line in the figure. Obsolescence is far slower than is normally assumed in the analysis of patent values.

Imperfect patent protection.—If a patent were to expire, this would allow entry without the payment of the license fee. Condition (10) would then no longer rule out multiple firms in some of the markets. Let us use the following shortcut: Let δ be the random rate at which the original owner of a patent right loses it permanently. Assume that, instead, someone else – a random person – inherits it so that license fees must still be paid for the right to use z. Then the demand-side is unaffected, and only the inventor suffers a loss. The bottom line in Figure 3 corresponds to a patent value that has a constant probability $\delta = .056$ of expiring and that therefore has an expected lifetime of 18 years, currently the maximum patent life in the U.S. For an
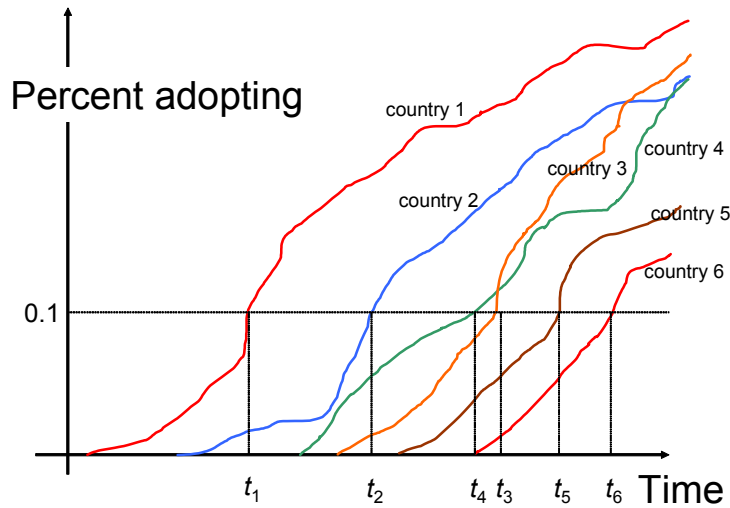
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Figure 4: AVERAGE DIFFUSION RATES AND THE CALCULATION OF THE t_i

arbitrary δ, the value of the patent right to the frontier z is

$$
\begin{aligned}
V(1) &= \int_0^T e^{-(r+\delta)t} p_t(1)\, dt \\
&= \gamma_0 \left(\frac{1 - e^{-(\eta+\delta)T}}{\eta + \delta} - \frac{1 - e^{-(\eta+\delta-g)T}}{\eta + \delta - g} z_{\min} \right).
\end{aligned}
$$

The rise in δ has only a level effect by reducing θ. Without transitional dynamics it cannot be given precisely, but we may conjecture that h would not grow any faster in the transition. If so, the level effect is the effect on $A z_{\max}/h_{\max} = \theta A$ which, in turn, is just the change in $\theta^{2-1/\alpha}$. We shall evaluate this policy change below.

4 Comparison to data

This section reports results from data on technological adoption; the data are described in Comin and Hobijn (2004). They cover 20 advanced countries and eleven technologies over the past two hundred years.[3] Table 1 reports t_i, defined as the average of the dates that the eleven technologies spread to ten percent of country i's population. Ten percent is low enough that nine of eleven technologies have reached it in all countries covered.[4] Figure 4 illustrates how the t_i were calculated.

[3] See the "Historical Cross-Country Technological Adoption: Dataset" at www.nber.org/data/

[4] The eleven technologies are private cars, radios, phones, television, personal computers, aviation passengers, telegraph, newspapers, mail, mobile phones, rail, and the telegraph.
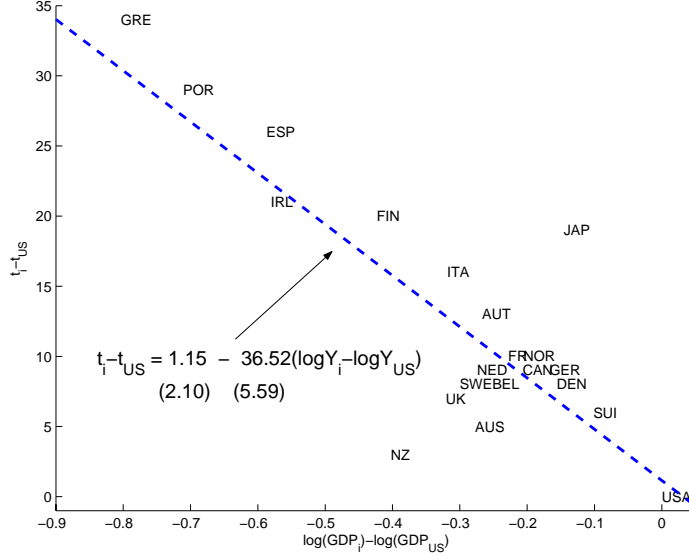
Figure 5: PLOT OF (23) AND THE DATA

In (22), replacing "max" by "USA" and "min" by "i", we have $\frac{Y_i}{Y_{USA}} = e^{-g(t_i - t_{USA})}$ from which we have

$$t_i - t_{USA} = -\frac{1}{g}\left(\ln Y_{US} - \ln Y_i\right). \tag{23}$$

A plot of the variables in the table is in Figure 5. The slope is negative and significant. The regression line should pass through the point $(0,0)$ which it does almost exactly – the constant does not differ significantly from zero. If countries were homogeneous in h, the regression's slope would, in theory, be $\frac{-1}{g}$ which, with $g = .015$, would be -67. But countries are not homogeneous: Table 5 of Sala-i-Martin (forthcoming) shows within-country inequality to be between 28% and 38% of inequality worldwide. Therefore the slope should have been $-\left(\frac{2}{3}\right)67 = -45$, and it does not differ significantly from that value.

World inequality far exceeds inequality in the C&H sample, as does T. Figure 6 reproduces's Figure 4A of Sala-i-Martin (2002). The model suggests that we should extrapolate the regression line as follows: Since $Y_{USA} - \ln Y_{min}^{world} \approx \ln 30 = 3.4$ while $\ln Y_{USA} - \ln Y_{min}^{C\&H} = 0.8$

$$\hat{T} = \left(\frac{\hat{d\tau}}{d\ln Y}\right)\frac{\ln Y_{USA} - \ln Y_{min}^{world}}{\ln Y_{USA} - \ln Y_{min}^{C\&H}} = (36.52)\frac{3.4}{0.8} = 154.5 \tag{24}$$

This may seem large, but there are plenty of examples of old technologies still in use. Many people in the world still have no access to electricity, a technology that was being commercially applied 120 years ago, and many still use animal power for plowing even though the tractor was commercialized by 1910.
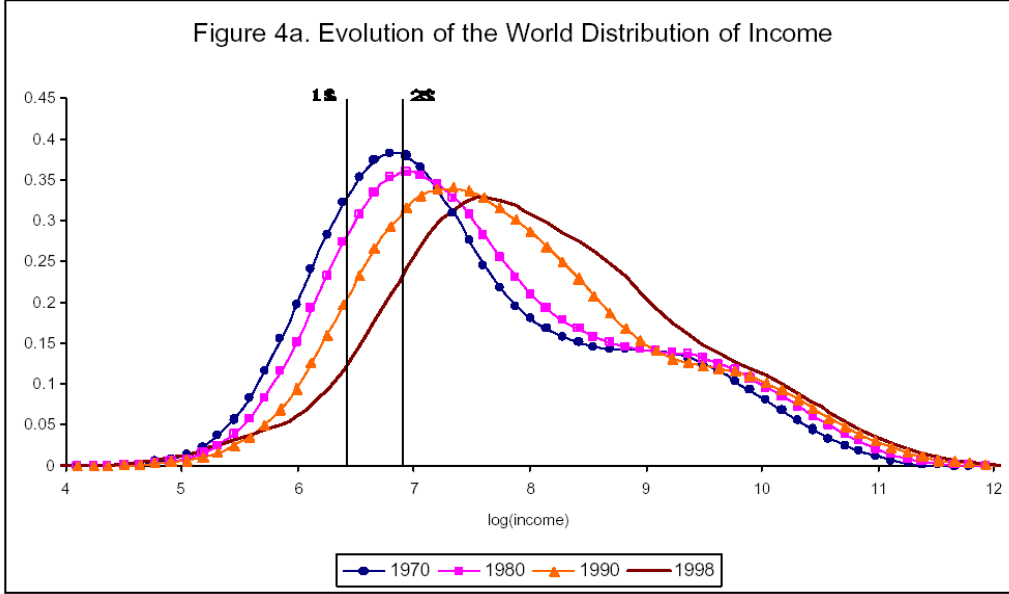
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Figure 6: Sala-i-Martin's Figure 4A

Inequality explained.—At the baseline values, the ratio in (22) of richest to poorest is 10.2. But when we apply this to countries, we must adjust for within-country inequality. Table 5 of Sala-i-Martin (2002) states that within-country inequality is roughly a third of world inequality, so the model can explain per-capita-income ratios of about 7:1

Success but partial.—The model succeeds only up to a point in explaining inequality – it gets perhaps 1/4 of it. It does not explain why incomes are stratified by country. And with its prediction that log incomes are uniformly distributed, it misses the skewness evident in Figure 6. Then there is the all-important parameter Δ on which it seems hard to get independent information. as follows: Combining (17) and (20) we end up with $\frac{1}{y}\frac{dy}{dt} = \frac{\Delta}{\alpha}N_t$. Let N_t^P denote U.S. patents issued at t, and let N_t^* be the HP-filtered version of N_t^P. De-trending is needed because patents increase over time whereas growth of y does not. Form a patent "stock" $\tilde{N}$ by the perpetual inventory method: $\tilde{N}_t = \frac{1-\mu}{1-\mu^{T-t}}\sum_{j=0}^{t}\mu^j N_t^*$. The regression $\ln y_{t+1} - \ln y_t = a\tilde{N}_t$ yields an estimate for a of 63.6 with a s.e. of 15.0. This estimate is way larger than one could ever reconcile with reasonable values of g and T via (). But the regression is mis-specified in that the estimated relation should hold across steady states and not in the time series; y is U.S. output and not world output, and N is U.S. patents, not patents world wide, so that the units are wrong. Moreover, patents have risen sharply in the '90s and surely are not proportional to N – if they were, g would have exploded.

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5 Policy experiments

To evaluate a couple of policies we need as realistic a benchmark version of the model as possible. The six endogenous variables are g, T, θ, u_P, u_R, and u_I.[5]

The restriction in (4) implies that the production function in (2) has increasing returns to scale of $1 + \beta = 1/\alpha$. The elasticity of demand is $-1/(1-\alpha)$. Thus (4) restricts these two magnitudes. Evidence in Klette and Griliches (1996, p. 344) is consistent with this restriction. Assume z and s are inputs that the econometrician measures. In terms of our notation, they estimate

$$1.06 \leq 1 + \beta \leq 1.1 \quad \text{and} \quad 6 \leq \frac{1}{1-\alpha} \leq 12.$$

If we maintain (4), the first set of inequalities holds for $0.909 \leq \alpha \leq 0.943$, whereas the second holds for $0.833 \leq \alpha \leq 0.917$. The midpoint of the region of overlap is $\alpha = 0.913$, and I shall use this value in the calibration. Returns to scale are hard to estimate, however, and many have estimated *decreasing* and not increasing returns. Such estimates are incompatible with (4) unless we assume that the econometrician does not observe z. But when a firm faces a downward-sloping demand curve, its output price falls as its output grows. Since firm-specific prices are usually unavailable, the firm's output growth is understated, and its returns-to-scale estimates are biased down.

Table 1 reports the baseline values of all the parameters, the endogenous variables, and comments on why they were picked.

Parameter	Reason for value chosen	Endog. varbl.	Reason for value chosen
$\rho = 0.0295$		$T = 154.5$	extrapolated via (24)
$\alpha = 0.913$	$\frac{1}{\alpha} = 1.095$ Gril.&Klette		
$\sigma = .6550$		$g = 0.015$	growth of output per head
$\lambda = 1$	only ratio θ/λ matters	$u_P = 0.5905$	
$\eta = 0.038$		$u_R = 0.0147$	$0.007 = $ R&D/Income
$\Delta = 2.31$	$Y_{\max}/Y_{\min} = 10$ (c.f. Prop. 5)	$u_I = 0.3948$	
		$r = 0.04$	

TABLE 1

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[5]The experiments will both entail level effects on output that work through the z/s ratio θ. In this, I shall assume that s is invariant to the policy, and that it is z that responds. This seems reasonable in that $g = \dot{h}/h$ depends on these policies only through the interest rate. The expression for A in (20) is misleading because it takes $z_{\max} = 1$ as given, in which case a rise in θ implies a lowering of h. The experiments we are about to perform assume the opposite; for fixed s, a rise in θ implies a higher z and higher output.
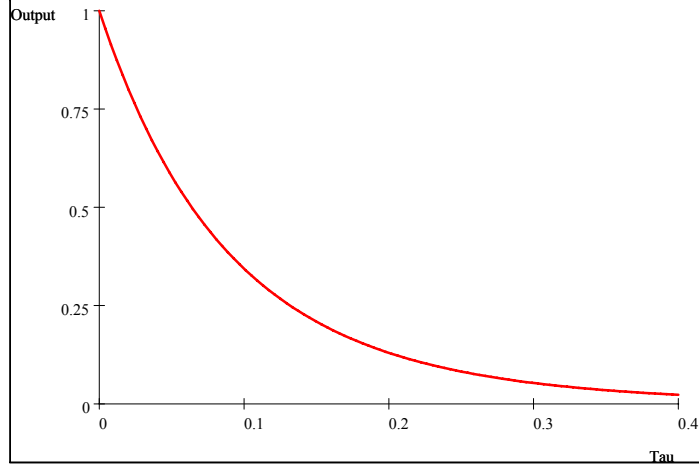
LEVEL EFFECTS OF τ

Taxes and Tariffs.—Taxes on a measure-zero subset of agents do not affect $p(z)$, or the rewards to research, so that g, T, and the values of the other endogenous variables remain the same. But income taxes are neutral whereas tariffs reduce the income of the taxed agents. (i) A *proportional income tax*, τ, on incomes of intermediate-goods producers change profits to $\pi(s) = (1-\tau)\max_z\{y^{1-\alpha}z^\alpha s^{1-\alpha} - p(z)\}$, and they do not affect the decision about z. But because costs of human-capital investment are all in the form of foregone earnings, $\dot{h}$ is unaffected as well, just as in Lucas (1988). (ii) *Tariffs* on the production of the final good imply no losses because profits there are zero. A proportional tax on technology, however, imply that $\pi(s) = \max_z\{y^{1-\alpha}z^\alpha s^{1-\alpha} - (1+\tau)p(z)\}$. The first-order condition for z now reads $\frac{\alpha}{1+\tau}\left(\frac{sy}{z}\right)^{1-\alpha} - p'(z) = 0$, so that

$$z = \frac{\theta s}{(1+\tau)^{1/(1-\alpha)}}.$$

These are only level effects, however; $\dot{h}/h$ stays the same. The level effects of τ plotted above are large, owing mainly to the high baseline value of α.

Intellectual property rights.—In the model, patents are infinitely lived, but in fact markets for technology are imperfect. Two measures of how well these markets work are (i) *Licensing revenues*: Firms recover only a fraction of R&D costs by selling or licensing their technology. As a percentage of R&D costs, royalty receipts (from abroad) in 2001 for patents, licenses, and copyrights were 64 (U.K.), 36 (Italy), 31 (Germany), 15 (U.S.), 11 (France) and 8 (Japan) (OECD 2004, tables 69-71); (ii) *International patenting*: Eaton and Kortum (Table 1) document that the U.S., the U.K., France, Germany and Japan patent abroad only about one fifth of the patents that they take out domestically, although it is probably those patents with the highest value that get patented abroad, and the distribution of patent values is known to be
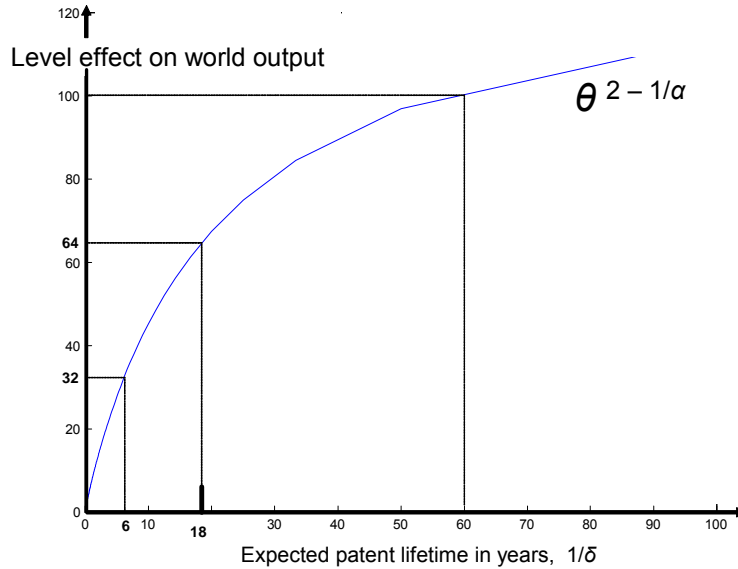
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Figure 7: Effect of patent protection on world output

highly skewed. It is hard to say how these numbers all translate into δ. But as δ rises and patent lifetime falls, output can fall dramatically. Reducing lifetime from 18 years (the current U.S. length, but perhaps a lot larger than effective worldwide protection) to 6 reduces output by a factor of 2.

6 Conclusion

Inequality arises in this model because at any time there are high-tech and low tech products, and because high-tech products are complements with human capital. Producers of high-tech products then invest more in human capital, and this produces inequality. As a result, world inequality depends on the rate at which products improve. The faster is this rate, the bigger is the technological asymmetry among products.

The product cycle is a symptom of comparative advantage at work: A well-functioning market for intellectual property leads to a maximum for world output given the available supply of technologies and skills. In a world with no license fees and patents, the model also says that inequality would not exist. But the common level of income would be many times smaller than average world income is today. Intellectual protection is, in fact, only partial and the model suggests that stronger enforcement would raise world output substantially.

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7   Appendix: Proof of Proposition 2

Let us ignore the term $y^{1-\alpha}$ which is common to all payoffs. Then (10) reads

$$\left(\theta s \left[s^\beta + s_0^\beta\right]\right)^{\alpha-1} (\theta s) s_0^\beta - \alpha\theta^\alpha (s - s_{\min}) \leq \theta^\alpha (\alpha s_{\min} + [1-\alpha] s_0),$$

which reduces to $\left(\theta s \left[s^\beta + s_0^\beta\right]\right)^{\alpha-1} (\theta s) s_0^\beta \leq \alpha\theta^\alpha s + \theta^\alpha (1-\alpha) s_0$ and then to $s^\alpha \left(s^\beta + s_0^\beta\right)^{\alpha-1} s_0^\beta \leq \alpha s + (1-\alpha) s_0$. Recalling (4), we find that both sides of the above inequality are homogeneous of degree 1 in (s, s_0). This means that is if the inequality holds at date 0 for s and s_0 in the interval $[s_{\min}, s_{\max}]$, it will hold for all s and s_0 in the interval $[e^{gt} s_{\min}, e^{gt} s_{\max}]$, and the latter is how the boundaries grow in steady state. Dividing by s we have

$$s^{\alpha-1} \left(s^\beta + s_0^\beta\right)^{\alpha-1} s_0^\beta \quad \leq \quad (1-\alpha)\frac{s_0}{s} + \alpha, \iff$$

$$\left[1 + \left(\frac{s_0}{s}\right)^\beta\right]^{\alpha-1} \left(\frac{s_0}{s}\right)^\beta \quad \leq \quad \alpha + (1-\alpha)\frac{s_0}{s}$$
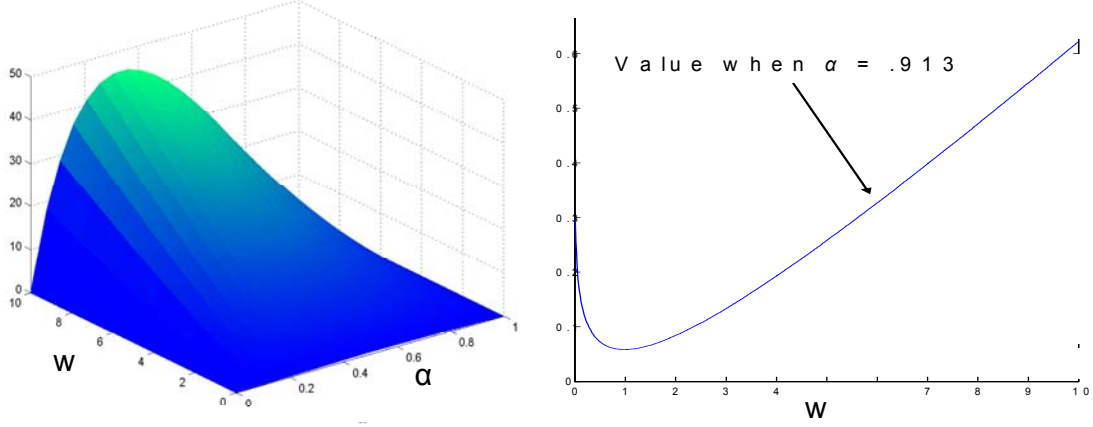
Figure 8: TWO PLOTS OF THE LEFT-HAND SIDE OF (25)

Because $\beta + (1 + \beta)(\alpha - 1) = 0$. Moreover, this must hold for all (s, s_0) between $s_{\min}$ and $s_{\max}$. Therefore it is equivalent to

$$\left(1 + w^\beta\right)^{\alpha-1} w^\beta \leq \alpha + (1 - \alpha) w$$

for $w \in \left[e^{-gT}, e^{gT}\right]$. But $\beta = (1 - \alpha)/\alpha$, therefore the condition is $\alpha + (1 - \alpha) w - \left(1 + w^{(1-\alpha)/\alpha}\right)^{\alpha-1} w^{(1-\alpha)/\alpha} \geq 0$, i.e., $\alpha + (1 - \alpha) w - \left(\frac{1+w^{(1-\alpha)/\alpha}}{w^{1/\alpha}}\right)^{\alpha-1} \geq 0$, i.e., $\alpha + (1 - \alpha) w - \left(w^{-1/\alpha} + w^{-1/\alpha+(1-\alpha)/\alpha}\right)^{\alpha-1} \geq 0$, i.e.,

$$\alpha + (1 - \alpha) w - \left(w^{-1/\alpha} + w^{-1}\right)^{\alpha-1} \geq 0. \tag{25}$$

Now (25) holds for all $\alpha \in (0, 1)$: At $w = 1$ the function reads $1 - \left(\frac{1}{2}\right)^{1-\alpha} \geq 0$. As $\alpha \to 0$ or as $\alpha \to 1$, the function converges to zero from above. This is all illustrated in Figure 8 for $\alpha \in (0, 1)$ and $w \in (0, 10)$.