ARTICLE IN PRESS YJETH:3881

Available online at www.sciencedirect.com
ScienceDirect

Journal of Economic Theory ••• (••••) •••–•••

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

Investment in vintage capital ☆

Boyan Jovanovic [a,*], Yuri Yatsenko [b]

[a] *Department of Economics, New York University, New York, NY 10012, USA*
[b] *School of Business, Houston Baptist University, 7502 Fondren, Houston, TX 77074, USA*

Received 27 September 2010; final version received 22 October 2010; accepted 27 October 2010

Abstract

We study an economy in which firms use labor and various vintages of capital in a CES production function for the final good. We explicitly solve for the investment in capital of a given vintage as a function of its age, and for the resulting stocks of capital. We show that for reasonable parameter values, inverted-U-shaped dynamics of investment and S-shaped dynamics for capital arise in equilibrium. We view the model as an explanation of intra-firm adoption lags, i.e., the observation that firms adopt innovations over time and not instantaneously.

JEL classification: D24; E22; O32

Keywords: Vintage capital models; CES function; General equilibrium; Technological change; Intra-firm adoption lags; Optimal control

1. Introduction

This volume is a welcome occasion to recall the contributions that David Cass made to the analysis of economies with heterogeneous capital. Growth models that feature many types of capital come in two forms: The putty-clay model of Johansen [24] and Arrow [2] and the vintage-capital model of Solow [39]. The models are often lumped together because they all have the feature that once a machine is constructed, its production characteristics – as described by the

☆ We thank the Kauffman Foundation for support, and participants of the FRB-Philadelphia & NBER Macroeconomics: Across Time and Space Conference (April 2010) the Editor and referees for comments.
* Corresponding author. Fax: +1 212 995 4186.
E-mail address: bj2@nyu.edu (B. Jovanovic).

0022-0531/$ – see front matter
doi:10.1016/j.jet.2010.10.017

Please cite this article in press as: B. Jovanovic, Y. Yatsenko, Investment in vintage capital, J. Econ. Theory (2010), doi:10.1016/j.jet.2010.10.017

machine's labor-input requirement, or by the number of its efficiency units – are fixed and cannot be changed *ex post*. The difference, however, is that the putty-clay models endogenize the quality of capital; in this sense they are models of endogenous technological change.

Cass and Stiglitz [9] construct a putty-clay model in which technological change is endogenous. In their model labor-intensive capital is cheaper to build. In the initial stages when capital of any type is scarce, firms invest in such technology. Over time capital labor ratio grows, and with it the equilibrium wage, and firms find it optimal to invest in more capital-intensive technology. Moreover, as the wage rises, the old labor-intensive capital is scrapped. The authors analyze the perfect-foresight equilibrium that features rich transitional dynamics. The analysis has a thoroughly modern feel to it, and with minor modifications of the model, the technological change could be made to persist even in the long run.

The present paper is motivated the observation that all the models discussed above including the Cass and Stiglitz [9] model entail no interaction in production among capital of different vintages. All investment flows into capital of the latest vintage, and none into any earlier vintage. A second generation of vintage-capital models such as Domar [16], Jorgenson [30], Greenwood, Hercowitz and Krusell [19], Xepapadeas and de Zeeuw [42], and Goetz, Hritonenko and Yatsenko [18] assumes that while capital of different vintages can participate in the same production process, the elasticity of substitution in production between capital of different vintages is infinite so that, again, all investment is in the latest-vintage capital where the efficiency of investment is highest.

The implication that all investment should flow into capital of the latest vintage is in conflict with experience. Old structures are refurbished, old machines are repaired, old workers are retrained. In other words, firms invest in old capital while also investing in new, more efficient capital. Evidently, old and new capitals interact and the productivity of one depends on the quantity of the other.

In this paper we study an economy in which firms face a CES production function whose inputs are labor and different vintages of capital. Ethier [17] introduced a production function that was an aggregate of intermediate goods, and one that has been used in growth models by Romer [38] and others. We suppose that some of these goods are durable, and that they are invented at different dates. That is, the heterogeneous durable goods are capital of different types, and the capital is of various vintages.

Such a production function has been estimated at various levels of aggregation. At the industry level, Boddy and Gort [6] estimate a production function that includes equipment and structures the average ages of which differ. At the firm level, Colombo and Mosconi [12] study the gradual adoption of various flexible-production and design-engineering technologies in the Italian metal-working industry, emphasizing that the technologies are complementary, that they are embodied in capital, and that they entail productivity growth as a result of learning by using.

Our model explains the staggered adoption of new technologies by firms, i.e., the fact that a firm usually adopts an innovation over time, not instantaneously. This phenomenon is sometimes referred to as *intra*-firm diffusion lags. It was first documented systematically by Mansfield [33, Table 1], who found that in the U.S. railroad industry, the time interval between 10 and 90% usage of diesel locomotives ranged from three to more than 14 years for firms in the U.S. railroad industry, with the median delay of 8–10 years. Similar within-firm delays occur in ten other U.S. industries by Romeo [37, Table 4]. Within firm adoption also tends to follow an S shape [23], with a firm's rate of spending on the technology in question peaks some years after investment in it begins.

Please cite this article in press as: B. Jovanovic, Y. Yatsenko, Investment in vintage capital, J. Econ. Theory (2010), doi:10.1016/j.jet.2010.10.017

The usual explanation of staggered intra-firm adoption of technology is that there are internal adjustment costs leading to protracted adoption, e.g. Stoneman [41] and Jensen [23]. One can view our model as a micro foundation for such adjustment costs. The production function in our model features a complementarity of the capital of different vintages and a concave learning curve of a new technology. The model can explain S-shaped adoption patterns emerge only if learning and complementarity are both present.

We solve the model completely; in other words, we solve analytically for each type of capital and for investment in that capital as a function of its age, and show that investment in old capital follows an S shape. We thereby establish analytically results that were established numerically by Chari and Hopenhayn [10], Cooley, Greenwood and Yorukoglu [14] and Atkeson and Kehoe [4]. The latter two papers assume, as do we, that the learning curve is a function of time. It remains to be seen whether similar results would emerge if learning depended on cumulative output as in Arrow [2] or Klenow [31], or if learning depended on investment and on the age distribution of the firm's employees as in Prescott and Boyd [35].

We also find that an economy in which technological progress is faster and the elasticity of substitution in production is higher, will have an age-distribution of capital that first-order dominates the age-distribution of capital in other economies. We also show that as the elasticity of substitution between different capital vintages becomes large, delays in learning cease to matter in the sense that the model becomes observationally equivalent to one in which there is no learning. These results are analytical and pertain to steady states only.[1]

In some vintage-capital models capital combines with a fixed resource such as labor (Johansen [24], Arrow [2], Jovanovic [25]), land and labor (Cooley, Greenwood and Yorukoglu [14]) or management and labor (Atkeson and Kehoe [4]), and in which that resource can work with only one technology, one type of capital, or one vintage of plant. In such models new capital must necessarily drive out the old since only a limited number of its vintages can physically be in use at any time. Our model imposes no such limitation; an unlimited number of vintages can coexist in production, although parameters are restricted so that no individual vintage is essential.

We confirm analytically the numerical result in Fig. 1 of Chari and Hopenhayn [10], which is that a decrease in the elasticity of substitution in production between the various vintages of capital delays investment in old capital. In particular, we solve for the age of a vintage at which peak investment occurs, and it is a decreasing function of the elasticity of substitution between old capital and new capital. Closer to our paper is the model of Kredler [32] which includes a continuum of vintages and a continuum of human-capital inputs in each technological vintage. In a related model Aruga [3] shows that when each vintage production function requires two complementary capital stocks that depreciate at different rates, investment in old capital is optimal when it allows to efficiently utilize the undepreciated stock of complementary capital. And in a related model without learning, Jovanovic [26] provides conditions for investment in old capital to be positive along a balanced-growth path. Such a characterization is quite limited for it says nothing about the investment profile.

For slow intra firm diffusion there are at least two other explanations. The first is that a technology improves over time or gets more user friendly as its manufacturers make it better and the capital embodying it cheaper. Combining this force with the diminishing marginal product of the capital in the firm will lead the firm to at first buy only that quantity of capital that it needs for

[1] Full dynamics in the vintage-capital model are hard to describe analytically, but some results are in Mitra, Ray and Roy [34], Boucekkine et al. [7,8], and Hritonenko and Yatsenko [20–22].

Please cite this article in press as: B. Jovanovic, Y. Yatsenko, Investment in vintage capital, J. Econ. Theory (2010), doi:10.1016/j.jet.2010.10.017

those tasks on which the improvements that the new technology can perform are the greatest; for the more marginal improvements, the firm will then wait and it will buy more of the capital only when its price has declined sufficiently. If the technology's price decline is steady, however, this implies a concave adoption path, not an S-shaped one. The second explanation pertains to situations in which a firm need to borrow to finance the adoption of the new technology. Clementi and Hopenhayn [11] show that when there are certain informational frictions and when the lender can commit to a contractual form, the optimal lending contract allows the firm's capital stock grows gradually. This may explain intra-firm diffusion lags among small firms, but not large firms that often finance investment from retained earnings.[2]

An alternative formulation is to assume that there are many goods entering the utility function or intermediates entering the final-goods production function, in the Spence–Dixit–Stiglitz tradition, with new goods being of higher quality than old goods. Stokey [40] and Young [43] have such models without capital, and Jovanovic [27] has only general capital. Another type of model is the quality-ladder model in which new products replace older versions; if new versions use version-specific capital, then the replacement of products influences the age distribution of capital – Acemoglu [1] discusses some of the issues.

The plan of the paper is as follows: Section 2 introduces and analyzes the model, Section 3 presents some simulations of the model, Section 4 briefly discusses some data issues, and Section 5 concludes. Some of the proofs are in Appendix A.

2. The model

The economy consists of a unit measure of agents each endowed with a unit of labor and with preferences

$$U = \int_0^\infty e^{-\rho t} \frac{C_t^{1-\eta}}{1-\eta}\, dt,$$

where C_t is consumption at date t. The agents receive income from wages and from the dividends of perfectly competitive firms that rent their labor and own their capital of various vintages.

Firms have access to constant-returns-to-scale production functions

$$y = K^\alpha N^{1-\alpha}, \tag{1}$$

where y is output of the only consumption good, N denotes labor services rented, and K denotes an aggregate of capital stocks that the firms own. That aggregate is given by the CES form

$$K_t = \left(\int_{-\infty}^{t} A_{t-v} (z_v k_{v,t})^\beta \, dv \right)^{1/\beta}, \tag{2}$$

in which $k_{v,t}$ is the amount of capital at date t embodying technology of vintage v, z_v is the unit efficiency of the capital of vintage v, A_{t-v} is the age-dependent "learning curve" for the capital

[2] Our model has a representative firm and as such, it does not explain *inter*-firm diffusion lags which exceed intra-firm diffusion lags. Early attempts to explain the inter-firm lags assume that some firms faced higher adoption costs than other firms. Chari and Hopenhayn [10] assume that there are technology-specific investments that induce some firms to stick to an older technology for a while. Jovanovic and Lach [28] assume that there are free-riding externalities to adoption that induce some firms to wait for other firms to adopt first so that they can reap the cost-reduction benefits. Jovanovic and MacDonald [29] assume that some firms are simply not aware of, or for some reason just do not have the ability to adopt a new technology, and so they adopt the technology later than other firms.

Please cite this article in press as: B. Jovanovic, Y. Yatsenko, Investment in vintage capital, J. Econ. Theory (2010), doi:10.1016/j.jet.2010.10.017

of vintage v, $t \in [0, \infty)$, $v \in (-\infty, t]$. The parameter $0 < \alpha \leqslant 1$ describes the return to scale and $0 < \beta \leqslant 1$ indexes substitution possibilities yielding the elasticity of substitution $\sigma = 1/(1-\beta)$.[3]

The case $0 < \beta < 1$, i.e., $\sigma > 1$ is the one that we judge to be realistic, because then no vintage of capital is essential: Positive output can be produced with any subset of vintages. We shall maintain this assumption throughout. Moreover, in contrast to the formulation that Ethier [17] and Romer [38] the production function (A) has constant returns and exhibits no preference for variety *per se*, and (B) treats inputs asymmetrically. Also, in contrast to the putty-clay models, factor proportions (including the capital-labor ratio) are variable *ex post*. In particular, in our model a unit of labor combines – in the *same* production function in Eq. (2) – with an ever larger number # of machine types.

Let x_t denote investment in new capital of vintage t and $u_{v,t}$ investment in old capital of vintage $v < t$ (of the age $t - v$). Let capital depreciate at the constant rate $\delta > 0$. Then the law of motion for capital of vintage v is $\frac{\partial k_{v,t}}{\partial t} = -\delta k_{v,t} + u_{v,t}$. This ordinary differential equation has the solution

$$k_{v,t} = e^{-\delta(t-v)} x_v + \int_v^t e^{-\delta(t-s)} u_{v,s}\, ds \tag{3}$$

for $v > 0$, and

$$k_{v,t} = e^{-\delta(t-v)} k_{v,0} + \int_0^t e^{-\delta(t-s)} u_{v,s}\, ds \tag{4}$$

for $-\infty < v < 0$, where $k_{v,0}$, $v \in (-\infty, 0]$, is a distribution of capital over past vintages at date zero that is given.

The firm solves the following profit maximization problem

$$\max_{u,x,N} \int_0^\infty e^{-rt} \left(y_t - w_t N_t - x_t - \int_{-\infty}^t u_{v,t}\, dv \right) dt, \quad x \geqslant 0,\ u \geqslant 0, \tag{5}$$

subject to (1), (2), (3), and (4) and subject to the initial conditions. The decision variables in (5) are x_t, the investments in new capital, $u_{v,t}$, investments in the various vintages of old capital, and labor N_t. The unknown capital amount $k_{v,t}$ and product output y_t are determined from (1)–(2) and (3)–(4) respectively for $0 < t < \infty$, $-\infty < v \leqslant t$.

2.1. Definition and standard properties of the equilibrium

The problem (1)–(5) entails no uncertainty so the solution of the model will be a perfect-foresight equilibrium for both firms and consumers. Wages must clear the labor market and interest rates must be such that representative consumer is happy to consume his wages and

[3] The function in (2) generalizes the traditional vintage aggregation functions of the form $K_t = \int_{-\infty}^t A_{t-v} z_v k_{v,t}\, dv$, which occurs at $\beta = 1$ (i.e., $\sigma = \infty$) and assumes that vintages of capital are perfect substitutes as, for instance, in Arrow [2] and Cass and Stiglitz [9, Eq. (2.3)]. Models that feature A_t are Cooley et al. [14] and Atkeson and Kehoe [4] who assume that A grows exogenously in each plant as a function of its age. Estimates of A are provided by Bahk and Gort [5].

Please cite this article in press as: B. Jovanovic, Y. Yatsenko, Investment in vintage capital, J. Econ. Theory (2010), doi:10.1016/j.jet.2010.10.017

dividends at each date. The analysis simplifies if we assume that technological change occurs at a constant rate, so that efficiency of capital vintages is exponential:

$$z_v = e^{\gamma v}, \quad \gamma > 0.$$

In this case, a constant-growth equilibrium has the following three well-known properties:

(1) First, the rate of interest, r, is constant, and the isoelastic utility function U implies that consumption must grow at a constant rate which we shall denote by g. In the consumer's optimal savings problem, it is known (e.g., Rebelo [36, p. 504]) that the relation between the rate at which the consumption grows and the rate of interest is

$$g = \frac{r-\rho}{\eta}. \tag{6}$$

(2) Second, the fraction of income invested is constant, and (2) and (3) imply that K_t grows at the rate $\gamma + g$. Moreover, the representative firm sets $N_t = 1$ so that $y_t = K_t^{\alpha}$. Taking logs of both sides of (1) and taking the time derivative yields the solution for the growth rate

$$g = \frac{\gamma\alpha}{1-\alpha}. \tag{7}$$

(3) Third, the labor market is frictionless and all workers are always employed at a common wage. In other words, the markets for labor clear at the wage sequence (w_t). The equilibrium wage at each date equals the marginal product of labor evaluated at the equilibrium capital-labor ratio at that date. Since $N_t = 1$, we have

$$w_t = (1-\alpha)y_t = (1-\alpha)K_t^{\alpha}. \tag{8}$$

2.2. *A firm's investment decision*

The firm faces no convex adjustment costs of capital, only a constant cost of buying each type of capital. Therefore we would expect the optimal policy to result in the firm setting the marginal product of each type of its capital to its user cost. Since the price of each type of capital is normalized to unity, the various vintages of capital all have the same user cost of $r + \delta$ per unit. Appendix A shows that at an interior solution for investment in vintage v (i.e., at $u_{v,t} > 0$), the marginal product of $k_{v,t}$ must indeed equal its user cost; that is,

$$\alpha z_v^{\beta} y_t^{(\alpha-\beta)/\alpha} A_{t-v} k_{v,t}^{\beta-1} = r + \delta, \quad 0 < v \leqslant t,\ 0 < t < \infty. \tag{9}$$

If the left-hand side of (9) is less than its right-hand side, then $u_{v,t} = 0$, which means that at date t the firm does not invest in capital of vintage v.

The aim of this paper is to (i) characterize the evolution of each type of capital as it ages, and (ii) derive the optimal investment policy $u_{v,t}$. If we can assume that the solution is always interior (and this assumption we shall return to), then (i) and (ii) are straightforward. It will turn out, however, that when σ is sufficiently large (i.e., when β is close to unity), $u_{v,t}$ becomes zero after some point.

2.2.1. *A diminishing rate of growth of A*

If we allow arbitrary learning functions A, then the interiority of the optimal investment decision will sometimes hold and sometimes not. However, we shall assume throughout that the

Please cite this article in press as: B. Jovanovic, Y. Yatsenko, Investment in vintage capital, J. Econ. Theory (2010), doi:10.1016/j.jet.2010.10.017

growth of learning is a diminishing function of a vintage's age. That is, if we define $s = t - v$ to denote the age of vintage-v capital, we shall assume that $\frac{A'_s}{A_s}$ is a monotonically declining function.[4] Then, the problem is well behaved in the sense that there is an interval of age of each capital stock on which investment is positive, and when a critical age is reached, the investment ceases and the capital is allowed to gradually depreciate to zero. That critical age may, however, be infinity in which case $\frac{A'_s}{A_s}$ investment in each vintage continues forever. Whether investment continues forever or not depends on whether the parameter κ defined by

$$\kappa = \frac{\gamma(\sigma - 1) - \delta - g}{\sigma} = \gamma\beta - \delta(1 - \beta) - \frac{\gamma\alpha}{1 - \alpha}(1 - \beta)$$

is positive or negative, and on whether $\frac{A'_s}{A_s}$ remains positive forever, as it will be, for example, if $A_s = 1 - e^{-\varphi s}$ or if $A_s = s^{\varphi}$.

Proposition 1. *Investment in a vintage continues* (*i.e.*, $u_{v,t} > 0$) *up to the point where*

$$\frac{A'_s}{A_s} = \kappa. \tag{P1}$$

If $\kappa < 0$, *so that the left-hand side of* (P1) *always exceeds the right-hand side, then the investment in a vintage continues forever. Capital* $k_{v,t}$ *peaks at the age* s *that satisfies*

$$\frac{A'_s}{A_s} = \gamma\frac{\sigma - 1}{\sigma}. \tag{P2}$$

We shall prove this proposition in the process of deriving the optimal capital stocks and investment levels. Before proving it, we make a few remarks about it.

First, investment continues beyond the point where capital peaks. This is illustrated in Fig. 1, which is drawn on the assumption that $\frac{A'(0)}{A(0)} > \kappa > 0$. Since the investment in old capital eventually ends, presumably this is the realistic case. Then k peaks at date t_1 whereas the investment ends at date $t_2 > t_1$. After the investment ceases, capital simply depreciates to zero, hence $k_{v,t} = k_{v,v+t_2}e^{-\delta(t-t_2)}$ for $t > t_2$.

Second, in accord with the intuition contained in Fig. 1 of Chari and Hopenhayn [10], the higher the elasticity of substitution is, the earlier investment stops, and the earlier the capital stock peaks. This is because the right-hand sides of (P1) and (P2) are both increasing in σ.

Third, $\kappa \to \gamma$ as $\sigma \to \infty$, and therefore t_1 and t_2 are both positive. Each generation of capital receives investment for a non-degenerate interval of time.

(i) *Optimal capital stocks.*

Solving (9) we obtain

$$k_{v,t} = \left(\frac{\alpha}{r + \delta} z_v^{\beta} y_t^{(\alpha-\beta)/\alpha} A_{t-v}\right)^{1/(1-\beta)}. \tag{10}$$

[4] We shall drop this assumption later in the context of Proposition 2 which hinges on an assumed learning curve in Eq. (21) in which A grows at a constant rate (which, however, is smaller than γ). In this case we can explicitly solve for the age distribution of capital, which we regard this as a limiting outcome of the case in which the growth of A diminishes with the age of the technology.

Please cite this article in press as: B. Jovanovic, Y. Yatsenko, Investment in vintage capital, J. Econ. Theory (2010), doi:10.1016/j.jet.2010.10.017

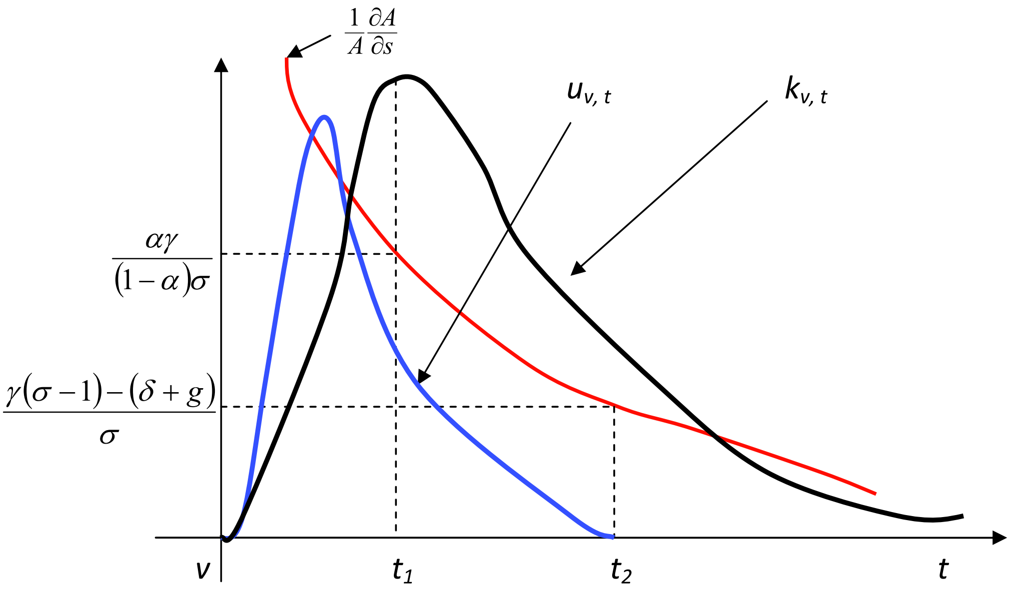

Fig. 1. Investment and capital as a function of age when $\kappa > 0$.

Now $y_t^{(\alpha-\beta)/\alpha} \propto \exp((\frac{\alpha-\beta}{\alpha})gt)$ and $z_v^{\beta} \propto \exp(\beta\gamma v) = \exp(\frac{\beta(1-\alpha)}{\alpha}gv)$. Substituting into (10) we see that, since $(\frac{\alpha-\beta}{\alpha})gt + \frac{\beta(1-\alpha)}{\alpha}gv = (1-\beta)gt - \frac{\beta(1-\alpha)}{\alpha}g(t-v) = (1-\beta)gt - \beta\gamma(t-v)$, (10) takes on the form

$$k_{v,t} = e^{gt}\chi_{t-v}, \tag{11}$$

where

$$\chi_s = \bar{k}e^{\frac{-\gamma\beta}{1-\beta}s}A_s^{\frac{1}{1-\beta}} \tag{12}$$

is a steady-state component that depends on the capital age $s = t - v$ only, and where the constant can be shown to equal

$$\bar{k} = \left(\frac{\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}}\left(\int_0^{\infty} e^{-\frac{\gamma\beta}{1-\beta}s}A_s^{\frac{1}{1-\beta}}\,ds\right)^{\frac{\alpha-\beta}{\beta(1-\alpha)}}. \tag{13}$$

Since output also grows at the rate g, the diffusion of capital relative to output is

$$\frac{k_{v,t}}{y_t} \propto \chi_{t-v}, \tag{14}$$

and it peaks at age $s = t - v$ at which $\partial\chi_{t-v}/\partial t = 0$, which implies that at the peak, $\frac{A_s'}{A_s} = \beta\gamma$. Since the elasticity of substitution is $\sigma = 1/(1-\beta)$, the peak occurs at the point where

$$\frac{A_s'}{A_s} = \gamma\frac{\sigma-1}{\sigma}, \tag{15}$$

as claimed in (P2) of Proposition 1.

Please cite this article in press as: B. Jovanovic, Y. Yatsenko, Investment in vintage capital, J. Econ. Theory (2010), doi:10.1016/j.jet.2010.10.017

(ii) *Optimal investment.*

New capital. Combining (11)–(13) with (3) at $v = t$, optimal investment in new capital is

$$x_t = k_{t,t} = \chi_0 e^{gt} = \bar{k} A_0^{\frac{1}{1-\beta}} e^{gt}. \tag{16}$$

Since this quantity grows as fast as income, the fraction of income invested in new capital is constant. If $A_0 = 0$ (as in the simulations below), there is no investment in new capital.

Old capital. Since $\frac{\partial k_{v,t}}{\partial t} = -\delta k_{v,t} + u_{v,t}$, we differentiate (11) and use (12) to find that

$$u_{v,t} = e^{gt} \tilde{u}_{t-v}, \tag{17}$$

where $\tilde{u}_{t-v}$ depends on the capital age $s = t - v$ only and is given by

$$\tilde{u}_s = \bar{k} e^{-\frac{\gamma\beta}{1-\beta}s} \left[\left(\delta + \frac{\gamma\alpha}{1-\alpha} - \frac{\gamma\beta}{1-\beta} \right) A_s^{\frac{1}{1-\beta}} + \frac{d}{ds} A_s^{\frac{1}{1-\beta}} \right]. \tag{18}$$

The exact condition for investment in the old, age-s capital to be positive is

$$\left(\delta + \frac{\gamma\alpha}{1-\alpha} - \frac{\gamma\beta}{1-\beta} \right) A_s^{\frac{1}{1-\beta}} \geqslant -\frac{d}{ds} A_s^{\frac{1}{1-\beta}}. \tag{19}$$

Letting $A_s^{\frac{1}{1-\beta}} \equiv m$, condition (19) is equivalent to $\frac{d\ln(m)}{ds} \geqslant \frac{\gamma\beta}{1-\beta} - \delta - \frac{\gamma\alpha}{1-\alpha}$. But $\frac{d\ln(m)}{ds} = \frac{1}{1-\beta}\frac{A_s'}{A_s}$, and so the investment-positivity condition reduces to $\frac{A_s'}{A_s} \geqslant \gamma\beta - \delta(1-\beta) - \frac{\gamma\alpha}{1-\alpha}(1-\beta) = \frac{\gamma(\sigma-1)-(\delta+g)}{\sigma} = \kappa$, as claimed in (P1) of Proposition 1.

If learning is bounded, then the left-hand side converges to zero for large s, as drawn in Fig. 1.

2.3. *Age distributions of capital and investment*

2.3.1. *The age distributions of capital and first-order dominance*

In order to compare the implications of our model to Chari and Hopenhayn [10] and specifically in terms of their first figure, we normalize the capital age-distribution in the simulation below by turning it into a density as follows

$$\hat{k}_s = \frac{\chi_s}{\int_0^\infty \chi_u \, du} = \frac{e^{-\frac{\gamma\beta}{1-\beta}s} A_s^{\frac{1}{1-\beta}}}{\int_0^\infty e^{-\frac{\gamma\beta}{1-\beta}u} A_u^{\frac{1}{1-\beta}} \, du} \tag{20}$$

and for the purposes of Proposition 2 we assume that the learning curve is

$$A_t = e^{-\theta t}, \tag{21}$$

where $\theta < \beta\gamma$. Then the distribution of capital age is exponential, that is

$$\hat{k}_s = m e^{-ms}, \quad \text{where } m = \frac{\beta\gamma - \theta}{1-\beta}.$$

The average age of capital is

$$T = \int_0^\infty t \hat{k}_t \, dt = \frac{1-\beta}{\beta\gamma - \theta}.$$

Please cite this article in press as: B. Jovanovic, Y. Yatsenko, Investment in vintage capital, J. Econ. Theory (2010), doi:10.1016/j.jet.2010.10.017

Thus we have

Proposition 2. *If the learning curve is as in* (21)*, then the age distribution of capital shifts to the left in the sense of first-order dominance when β or γ rise or when θ falls.*

As $\beta \to 1$, the average age of capital, T, converges to zero, and as $\theta \to \beta\gamma$ it converges to infinity, because it then becomes optimal to invest for ever in the same vintage.

2.4. When vintages are perfect substitutes: the special case $\sigma = \infty$

When different capital vintages are perfect substitutes except that the later vintages are of higher quality as Greenwood et al. [19] assume, then in the absence of learning effects (so that $A_s \equiv 1$), all investment flows into the latest-vintage capital and the old capital is simply left to depreciate, with the undepreciated portion remaining in use forever.

To analyze this issue, we assume the learning function is

$$A_s = 1 - e^{-\varphi s}, \quad \varphi > 0. \tag{22}$$

This also is the functional form that we shall also use in the simulations that follow below. As σ and ϕ both become large, all investment goes into the latest vintage.

Suppose, however, that $\sigma \to \infty$, but that the learning parameter ϕ remains finite, $\phi < \infty$. A related, observationally equivalent result then obtains: Almost all investment flows into that vintage in which investment creates the largest number of efficiency units of capital. A vintage does not receive investment immediately. Rather, there is a delay, D, and then the vintage receives (virtually) all the investment that it will ever get. We can show that, at $\sigma = \infty$, the formula for the delay is

$$D = \arg\max_s e^{-\gamma s}\left(1 - e^{-\varphi s}\right) = \frac{1}{\varphi}\ln\left(1 + \frac{\varphi}{\gamma}\right),$$

and, as ϕ gets large, D indeed converges to zero, so that our model with imperfect substitutes approaches the vintage model with perfect substitution and no learning that Greenwood et al. [19] consider. This result is illustrated numerically in Fig. 5.

The upshot is that as $\sigma \to \infty$ the model with learning becomes equivalent to a model without learning in which, instead of being $e^{\gamma t}$, the technology parameter is $Ae^{\gamma t}$, where

$$A = e^{-\gamma D}\left(1 - e^{-\varphi D}\right).$$

Thus, in models with perfect (and almost perfect) substitution among vintages, learning matters only in that it reduces the level of efficiency of the capital introduced at each date, with all other implications of the model being equivalent to a model with no learning. Of course, if one can observe the age of a technology, then one would find an extreme S-shaped diffusion of technology at $\sigma = \infty$: zero up to age D, and then all the adoption occurring exactly at age D, followed by no further adoption.

The observational equivalence referred to above is true for an outside observer, but not to the representative agent of the economy we are describing. When the time-path of technology is held constant, learning lags make the agent worse off. To an outside observer, however, the state of technology is not identified separately from D. Only its state corresponding to the time parameter $t - D$ is identified.[5]

3. Predicted and actual technological diffusion lags

This section compares the predicted diffusion delays to some evidence. The within-firm delay is a lot smaller than the between-firm delay and therefore we would expect our model to explain only a small part of aggregate diffusion delays. This turns out to be true.

3.1. Parameter choice

3.1.1. Learning parameters

Bahk and Gort [5, Sec. IV] find that most capital learning is over by the fifth or sixth year of the capital's age. Assuming that exactly 95 percent of the learning occurs by the end of the fifth year, the equation $1 - e^{-5\phi} = 0.95$ must hold, and this gives us the value $\phi = 0.6$ for A_s in (22).

3.1.2. Share of capital and embodied technological progress

Since real income per head tends to grow at roughly 1.5 percent per year, Eq. (7) indicates that

$$\frac{\gamma\alpha}{1-\alpha} = 0.015.$$

This is a joint restriction on α and γ. If we assume that K denotes a broad capital aggregate and that L denotes raw labor, then perhaps a value of $\alpha = 0.667$ is reasonable. In that case we must have $\gamma = 0.0075$.

3.1.3. The elasticity of substitution

We shall follow Chari and Hopenhayn [10] and entertain various values of β between zero and one (that imply values of σ between one and infinity).

The parameter $\sigma = 1/(1-\beta)$ is not precisely estimated in the literature, and its appropriate value presumably depends on the degree of aggregation of capital. The higher the aggregation, the lower σ should be. Boddy and Gort [6] estimate a production function in which capital is disaggregated into equipment and structures, and the average age of equipment is considerably smaller than the average age of structures. In that sense they estimate a production function with two broad aggregates of capital of different vintages, and their estimate of σ of 1.71 implies a value for β of 0.42, which should perhaps be viewed as a lower bound for β.

3.1.4. The utility-function parameters

The remaining parameters are the rate of utility discount, ρ, and the utility-curvature parameter η. They determine the rate of interest in (6). Since these parameters do not enter the other equations and the simulations, and since we are not sure what interest rate to apply to the discounting of the typical stream of incomes that investment in capital yields, we shall leave these two parameters unspecified.

[5] Others have noted, namely, that under some conditions one cannot identify age effects and vintage effects, at least not in aggregate data with a representative firm. With heterogeneous users identification is, under some conditions, possible (Coulson and McMillen [15]).

Please cite this article in press as: B. Jovanovic, Y. Yatsenko, Investment in vintage capital, J. Econ. Theory (2010), doi:10.1016/j.jet.2010.10.017

Table 1
Parameter values used in the simulations.

α	β	γ	δ	ϕ
0.667	$0 < \beta < 1$	0.0075	0.075	0.6

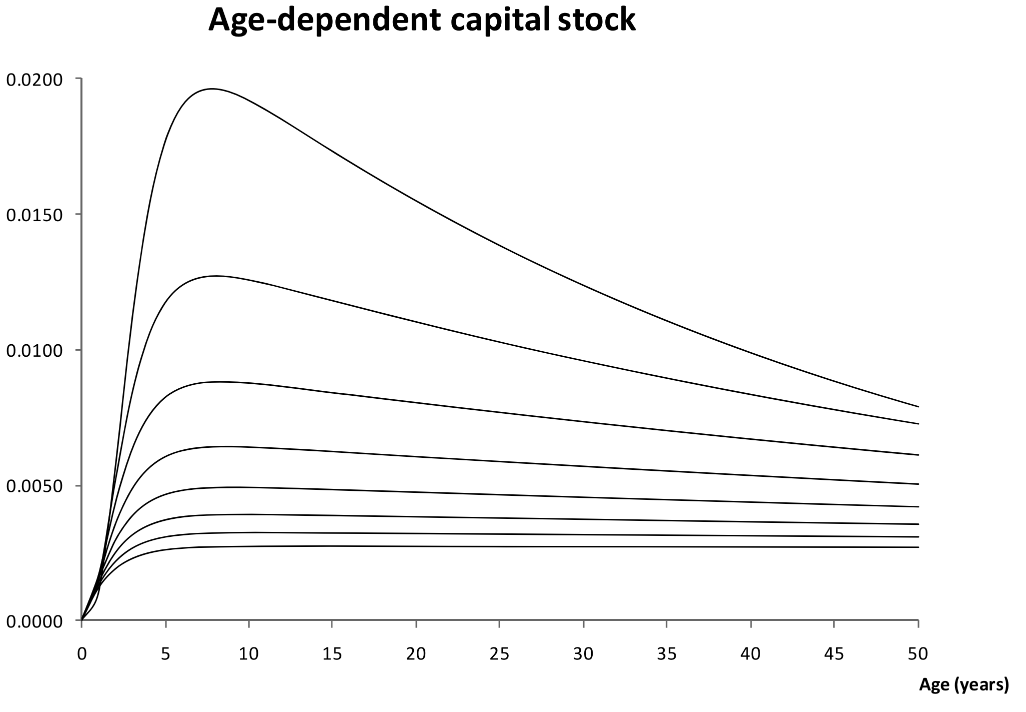

Fig. 2. Simulated age-distribution of capital for $\beta = 0.05, 0.15, 0.25, 0.35, 0.45, 0.55, 0.65, 0.75$.

The rate of depreciation is typically taken to be around 0.075, and so we shall assume this value for δ. Thus, we may summarize our parameter-value choices in Table 1.

Finally, the parameter κ in Proposition 1 ranges from -0.09 when $\beta = 0$ (so that $\sigma = 1$), to 0.0075 when $\beta = 1$ (so that $\sigma = \infty$).

3.2. Simulations

We now display some properties of the model graphically, focusing on the age distribution of capital and on the properties of the investment profile into a particular capital vintage as a function of the age of that vintage. To keep capital and investment in the same units, we apply the same normalization to the age-profile of investment in old capital:

$$\hat{u}_s = \frac{\tilde{u}_s}{\int_0^\infty \chi_u \, du}.$$

Fig. 2 plots the age distribution of capital for selected values of β between zero and 0.75.

In Fig. 2, the curves must eventually cross since the area under them is the same. These crossings do not show up in Fig. 2 because they occur after the technologies are 50 years old. The

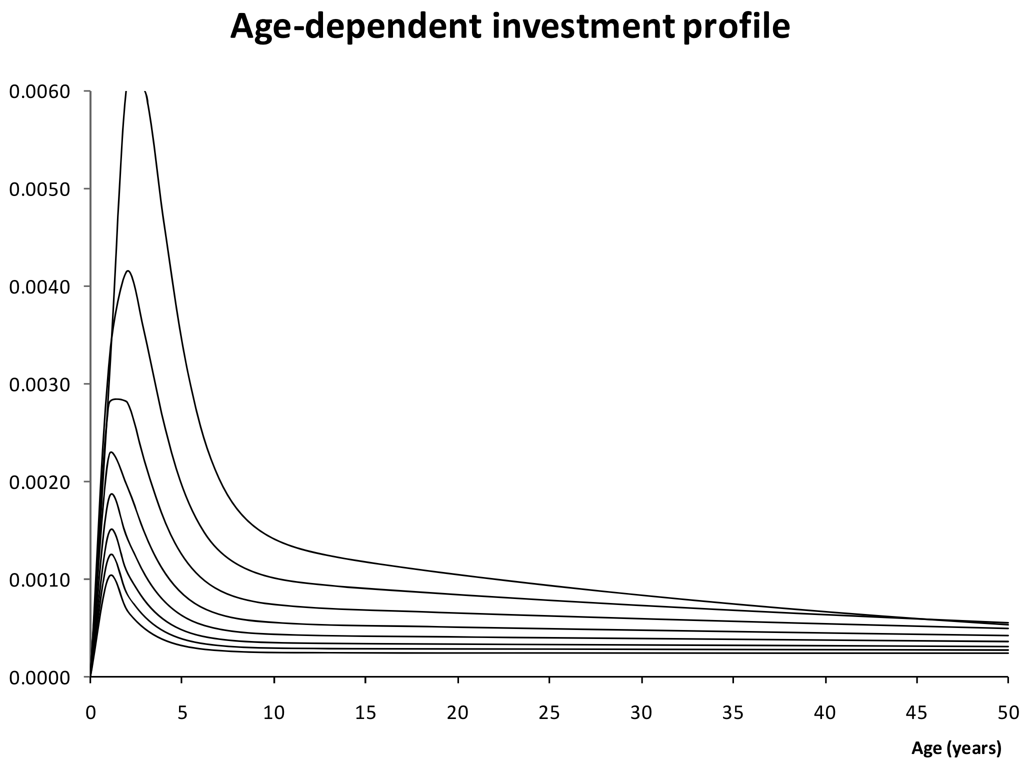

Fig. 3. Investment in capital by age for $\beta = 0.05, 0.15, 0.25, 0.35, 0.45, 0.55, 0.65, 0.75$.

high-β densities peak early and then decay fairly rapidly. The low-β densities peak even earlier, but they do not decay as rapidly, and eventually overtake the densities pertaining to values of β that are higher.

Fig. 3 plots the corresponding densities of investment which peak around the inflection points of the densities of capital.

Figs. 4 and 5 deal with values β close to unity and, consequently, with higher elasticities of substitution. The densities all cross within the first 50 years of the lives of the technologies.

3.2.1. S-shaped diffusion of technology

Both the capital stock and investment have an initial convex portion and, of course, both distributions eventually reach a peak, so that both capital and investment have an initial S shape. This happens because our simulations assume that learning takes the form given by Eq. (22), which implies that $A_0 = 0$. This low initial level of a technology's productivity discourages investment in that technology early on in its life. As mentioned in the above section on perfectly-substitutable vintages, as $\beta \to 1$ and $\sigma \to \infty$, the investment converges to a Dirac-type delta function located at the point

$$D = \frac{1}{\varphi} \ln\left(1 + \frac{\varphi}{\gamma}\right) = \frac{1}{0.06} \ln\left(1 + \frac{0.06}{0.0075}\right) = 7.32.$$

The consequences of this for the age distribution of capital are shown his is shown in Fig. 4; as β rises, the left tail of the age distribution becomes thinner and thinner, converging to zero at ages

Please cite this article in press as: B. Jovanovic, Y. Yatsenko, Investment in vintage capital, J. Econ. Theory (2010), doi:10.1016/j.jet.2010.10.017

Fig. 4. Simulated age-distribution of capital for $\beta = 0.86, 0.87, 0.88, 0.89, 0.9, 0.91, 0.92, 0.93, 0.94, 0.95$.

smaller than D (the age at which investment begins in the limit). The right tail does not converge to zero because, although all investment in the limit ceases at age D, the capital created at that time takes many periods to depreciate.

The pattern of investment as a function of technology age is plotted in Fig. 5, which shows clearly that as β rises, the distribution of capital becomes ever more spiked, attaining the Dirac distribution as $\beta \to 1$.

Fig. 6 is the simulated counterpart of Fig. 1 which was drawn schematically. Two versions are presented, one for $\beta = 0.9$ and the other for $\beta = 0.95$. The higher value of β leads to a more concentrated or "spiked" investment profile, and a correspondingly tighter distribution of capital. Specifically, the black solid lines depict the case where $\beta = 0.95$. The lower bell-shaped curve is the investment rate, and the investment ceases when the technology is 9 years old. The peak of the capital-stock distribution occurs slightly before that and the capital practically disappears after the age of 45 years. This is the case that corresponds most closely to the patterns depicted in Fig. 1. The two gray bell-shaped curves represent the distributions of capital and investment when $\beta = 0.90$, and in this case investment continues forever, but becomes quite small beyond age 15.

3.3. Comparison to data

A contrast between the calibrated version and the facts is that broad technologies diffuse more slowly. Peak investment is predicted to be somewhere between year 5 and year 10 following a technology's introduction, and this fits well the evidence in Table 1 of Mansfield [33] for the

Please cite this article in press as: B. Jovanovic, Y. Yatsenko, Investment in vintage capital, J. Econ. Theory (2010), doi:10.1016/j.jet.2010.10.017

Fig. 5. Investment in capital for $\beta = 0.86, 0.87, 0.88, 0.89, 0.9, 0.91, 0.92, 0.93, 0.94, 0.95$.

delays between 10% and 90% intra-firm diffusion, and the evidence in Table 4 of Romeo [37] for evidence for delays between 10% and 60% intra-firm diffusion. On this score, then, the model fits well if the elasticity of substitution is high. At lower elasticities of substitution such as those used for the simulations in Figs. 2 and 3, it takes far too long to reach 90% adoption rates, so that the model overpredicts the intra-firm delays.

We stress that the model does not aim to explain *inter*-firm diffusion lags. These are far longer, especially if we include firms located in different countries, as Comin and Hobijn [13] have shown.

4. Conclusion

This paper has analyzed a tractable growth model of vintage capital where the representative firm faces a CES production function. The model yields rich dynamics that none the less admit an exact and simple solution. The paper has focused on the role played by learning and by the elasticity of substitution among capital vintages along a steady state trajectory. It has shown that an economy in which technological progress is faster, or the elasticity of substitution in production is higher, will have an age-distribution of capital that first-order dominates the age-distribution of capital in other economies. It has shown, further, that in a sense, learning is irrelevant for the observational properties of the vintage-capital model when the elasticity of substitution becomes large.

Please cite this article in press as: B. Jovanovic, Y. Yatsenko, Investment in vintage capital, J. Econ. Theory (2010), doi:10.1016/j.jet.2010.10.017

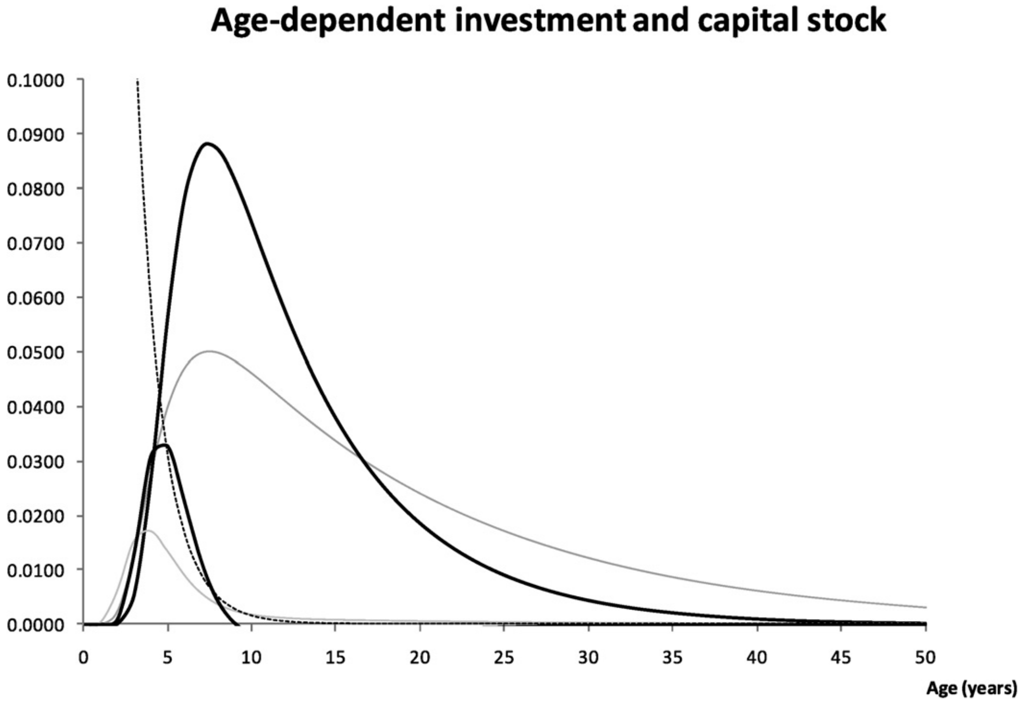

Fig. 6. Age-distribution of capital and investment for $\beta = 0.95$ ($\kappa = 0.00262$, black solid lines) and for $\beta = 0.9$ ($\kappa = -0.00225$, gray lines). The dotted line is A'_s/A_s.

Several other potential vintage-related issues can be studied in our framework. They include R&D investment, human capital, energy saving technologies, and aggregate dynamics, including the global stability of the steady-state growth path. Finally, we have studied a multi-capital CES production function in the context of a vintage-capital model in which technological change is exogenous. One could, instead, introduce such a production function into a putty-clay model of the type studied by David Cass and Joseph Stiglitz.

Appendix A

To derive necessary conditions for the firm optimization problem defined in (5), we apply the classic Lagrange multiplier method adjusted to vintage capital models in Hritonenko and Yatsenko [20,21]. Below we use the standard notations $f(t) = f_t$ and $g(v, t) = g_{vt}$ for all involved functions.

We consider a representative firm and assume that its employment remains at its equilibrium level $N(t) = 1$ at all dates. This is an equilibrium constraint that the price-taking firm does not face; rather it treats the wage as exogenous to its choice of K. We concentrate on the firm investment decisions $x(t)$ and $u(v, t)$.

The Lagrangian of the firm problem (1)–(5) with taking the optimal equilibrium wage (8) into account can be written as

Please cite this article in press as: B. Jovanovic, Y. Yatsenko, Investment in vintage capital, J. Econ. Theory (2010), doi:10.1016/j.jet.2010.10.017

$$L=\int_0^\infty e^{-rt}\Biggl\{K^\alpha(t)-x(t)-\int_{-\infty}^t u(v,t)\,dv$$
$$+\int_0^t \lambda(v,t)\Biggl(e^{-\delta(t-v)}x(v)+\int_v^t e^{-\delta(t-s)}u(v,s)\,ds-k(v,t)\Biggr)dv$$
$$+\int_{-\infty}^0 \lambda(v,t)\Biggl(e^{-\delta(t-v)}k_0(v)+\int_0^t e^{-\delta(t-s)}u(v,s)\,ds-k(v,t)\Biggr)dv\Biggr\}dt,$$

where $\lambda(v,t)$ is the unknown Lagrange multiplier function for the constraints (3) and (4). Using small variations $\delta u(v,t)$, $\delta x(t)$, $\delta y(t)$, and $\delta k(v,t)$ of the controls u, x, y, and k, we obtain that the corresponding increment δL of functional L is

$$\delta L\approx\int_0^\infty e^{-rt}\Biggl\{\delta y(t)-\delta x(t)-\int_{-\infty}^t \delta u(v,t)\,dv$$
$$+\int_{-\infty}^0 \lambda(v,t)\Biggl(\int_0^t e^{-\delta(t-s)}\delta u(v,s)\,ds-\delta k(v,t)\Biggr)dv$$
$$+\int_0^t \lambda(v,t)\Biggl(e^{-\delta(t-v)}\delta x(v)+\int_v^t e^{-\delta(t-s)}\delta u(v,s)\,ds-\delta k(v,t)\Biggr)dv\Biggr\}dt, \tag{A.1}$$

where, by Eqs. (1) and (2),

$$\delta y(t)=\alpha\Biggl(\int_{-\infty}^t A(t-v)z^\beta(v)k^\beta(v,t)\,dv\Biggr)^{(\alpha-\beta)/\beta}$$
$$\times\int_{-\infty}^t A(t-v)z^\beta(v)k^{\beta-1}(v,t)\delta k(v,t)\,dv.$$

Next, interchanging limits of integration in the double integrals and making other routine transformations, we represent (A.1) as:

$$\delta L\approx\int_0^\infty\Biggl[\int_t^\infty e^{-rs-\delta(s-t)}\lambda(t,s)\,ds-e^{-rt}\Biggr]\delta x(t)\,dt$$
$$+\int_0^\infty\int_{-\infty}^t\Biggl[\int_t^\infty e^{-rs-\delta(s-t)}\lambda(v,s)\,ds-e^{-rt}\Biggr]\delta u(v,t)\,dv\,dt$$
$$+\int_0^\infty\int_{-\infty}^t e^{-rt}\bigl[\alpha y^{(\alpha-\beta)/\beta}(t)A(t-v)z^\beta(v)k^{\beta-1}(v,t)-\lambda(v,t)\bigr]\delta k(v,t)\,dv\,dt. \tag{A.2}$$

Setting the coefficients at $\delta x(t)$, $\delta u(v,t)$, and $\delta k(v,t)$ in (A.2) equal to zero and excluding the dual variable $\lambda(v,t)$, we obtain the first-order extremum condition for an interior solution $u > 0$, $x > 0$, namely

$$1 = \alpha z^{\beta}(v) \int_t^{\infty} e^{-(r+\delta)(s-t)} y^{(\alpha-\beta)/\alpha}(s) A(s-v) k^{\beta-1}(v,s)\, ds$$

or

$$\alpha z^{\beta}(v) \int_t^{\infty} e^{(r+\delta)s} y^{(\alpha-\beta)/\alpha}(s) A(s-v) k^{\beta-1}(v,s)\, ds = e^{-(r+\delta)t}, \quad 0 < v \leqslant t,\ 0 < t < \infty. \tag{A.3}$$

Equality (A.3) at $v < t$ delivers the extremum condition for interior $u(v,t) > 0$, and the condition for interior $x(t) > 0$ at $v = t$. Differentiation of (A.3) with respect to t gives the necessary condition (9) represented in the text.

References

[1] D. Acemoglu, Labor- and capital-augmenting technical change, J. European Econ. Assoc. 1 (2003) 1–37.
[2] K. Arrow, The economic implications of learning by doing, Rev. Econ. Stud. 29 (1962) 155–173.
[3] O. Aruga, Conventional or new? Optimal investment allocation across vintages of technology, Ministry of Education, Japan, 2007.
[4] A. Atkeson, P. Kehoe, Modeling and measuring organization capital, J. Polit. Economy 113 (2005) 1026–1053.
[5] B.-H. Bahk, M. Gort, Decomposing learning by doing in new plants, J. Polit. Economy 101 (1993) 561–583.
[6] R. Boddy, M. Gort, The substitution of capital for capital, Rev. Econ. Statist. 53 (1971) 179–188.
[7] R. Boucekkine, M. Germain, O. Licandro, Replacement echoes in the vintage capital growth model, J. Econ. Theory 74 (1997) 333–348.
[8] R. Boucekkine, F. del Rio, O. Licandro, Endogenous vs. exogenously driven fluctuations in vintage capital models, J. Econ. Theory 88 (1999) 161–187.
[9] D. Cass, J. Stiglitz, The Implications of alternative saving and expectations hypotheses for choices of technique and patterns of growth, J. Polit. Economy 77 (1969) 586–627.
[10] V.V. Chari, H. Hopenhayn, Vintage human capital, J. Polit. Economy 99 (1991) 1142–1165.
[11] G.L. Clementi, H. Hopenhayn, A theory of financing constraints and firm dynamics, Quart. J. Econ. 121 (2006) 229–265.
[12] M.G. Colombo, R. Mosconi, Complementarity and cumulative learning effects in the early diffusion of multiple technologies, J. Ind. Econ. 43 (1995) 13–48.
[13] D. Comin, B. Hobijn, Cross-country technology adoption: making the theories face the facts, J. Monet. Econ. 51 (2004) 39–53.
[14] T. Cooley, J. Greenwood, M. Yorukoglu, The replacement problem, J. Monet. Econ. 40 (1997) 457–499.
[15] E. Coulson, D.P. McMillen, Estimating time, age and vintage effects in housing prices, J. Housing Econ. 17 (2008) 138–151.
[16] E. Domar, Total productivity and the quality of capital, J. Polit. Economy 71 (1963) 586–588.
[17] W. Ethier, National and international returns to scale in the modern theory of international trade, Amer. Econ. Rev. 72 (1982) 389–405.
[18] R. Goetz, N. Hritonenko, Y. Yatsenko, The optimal economic lifetime of vintage capital in the presence of operating cost, technological progress, and learning, J. Econ. Dynam. Control 32 (2008) 3032–3053.
[19] J. Greenwood, Z. Hercowitz, P. Krusell, Long-run implications of investment-specific technological change, Amer. Econ. Rev. 87 (1997) 342–362.
[20] N. Hritonenko, Y. Yatsenko, Modeling and Optimization of the Lifetime of Technologies, Kluwer Academic Publishers, Dordrecht, 1996.
[21] N. Hritonenko, Y. Yatsenko, Turnpike and optimal trajectories in integral dynamic models with endogenous delay, J. Optim. Theory Appl. 127 (2005) 109–127.

Please cite this article in press as: B. Jovanovic, Y. Yatsenko, Investment in vintage capital, J. Econ. Theory (2010), doi:10.1016/j.jet.2010.10.017

[22] N. Hritonenko, Y. Yatsenko, Anticipation echoes in vintage capital models, Math. Comput. Modelling 48 (2008) 734–748.
[23] R. Jensen, Strategic Intrafirm innovation adoption and diffusion, Southern Econ. J. 68 (2001) 120–132.
[24] L. Johansen, Substitution versus fixed proportions coefficients in the theory of economic growth: a synthesis, Econometrica 27 (1959) 157–176.
[25] B. Jovanovic, Vintage capital and inequality, Rev. Econ. Dynam. 1 (1998) 497–530.
[26] B. Jovanovic, When should firms invest in old capital, Int. J. Econ. Theory 5 (2009) 107–123.
[27] B. Jovanovic, The technology cycle and inequality, Rev. Econ. Stud. 76 (2009) 707–729.
[28] B. Jovanovic, S. Lach, Entry, exit and diffusion with learning by doing, Amer. Econ. Rev. 79 (1989) 690–699.
[29] B. Jovanovic, G. MacDonald, Competitive diffusion, J. Polit. Economy 102 (1994) 24–52.
[30] D. Jorgenson, The embodiment hypothesis, J. Polit. Economy 74 (1966) 1–17.
[31] P. Klenow, Learning curves and the cyclical behavior of manufacturing industries, Rev. Econ. Dynam. 1 (1998) 531–550.
[32] M. Kredler, Experience vs. obsolescence: A vintage-human-capital model, Univ. Carlos III, Madrid, 2008.
[33] E. Mansfield, The speed of response of firms to new techniques, Quart. J. Econ. 77 (1963) 290–311.
[34] T. Mitra, D. Ray, R. Roy, The economics of orchards: An exercise in point-input, flow-output capital theory, J. Econ. Theory 53 (1991) 12–50.
[35] E.C. Prescott, J. Boyd, Dynamic Coalitions: Engines of growth, Amer. Econ. Rev. 77 (1987) 63–67.
[36] S. Rebelo, Long-run policy analysis and long-run growth, J. Polit. Economy 99 (1991) 500–521.
[37] A. Romeo, Interindustry and interfirm differences in the rate of diffusion of an innovation, Rev. Econ. Statist. 57 (1975) 311–319.
[38] P. Romer, Endogenous technological change, J. Polit. Economy 98 (1990) 71–102.
[39] R. Solow, Investment and technological progress, in: K. Arrow, S. Karlin, P. Suppe (Eds.), Mathematical Methods in the Social Sciences, Stanford University Press, Stanford, CA, 1960.
[40] N. Stokey, Learning by doing and the introduction of new Goods, J. Polit. Economy 96 (1988) 701–717.
[41] P. Stoneman, Intra firm diffusion, bayesian learning, and profitability, Econ. J. 91 (1981) 375–388.
[42] A. Xepapadeas, A. de Zeeuw, Environmental policy and competitiveness: The Porter hypothesis and the composition of capital, J. Environ. Econ. Manage. 37 (1999) 165–182.
[43] A. Young, Invention and bounded learning by doing, J. Polit. Economy 101 (1993) 443–472.

Please cite this article in press as: B. Jovanovic, Y. Yatsenko, Investment in vintage capital, J. Econ. Theory (2010), doi:10.1016/j.jet.2010.10.017