Aspirations and Inequality[1]

Garance Genicot
Georgetown University

Debraj Ray
New York University and University of Warwick

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Abstract

This paper develops a theory of socially determined *aspirations* as reference points for the individuals. In this theory, society-wide economic outcomes shape individual aspirations, which affect the investment incentives of individuals. Through its impact on investments, individual aspirations in turn affect ambient social outcomes. We explore this two-way link in settings in which aspirations and income (and the *distribution* of income) evolve jointly. In particular, we explore the relationship between aspirations, growth and widening inequality. Our model captures both the inspiration and the potential frustrations that can result from higher aspirations.

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1. Introduction

In the 2014 general elections of India, the incumbent United Progressive Alliance, led by the Congress Party, was handed a resounding defeat by the National Democratic Alliance, headed by the Bharatiya Janata Party (BJP). The BJP alone won over half of all the contested seats, the first time since 1984 that a party had taken enough seats to govern on its own. Yet in the decade (2004-2013) that the United Progressive Alliance governed India — over two terms — Indian GDP per capita grew at the impressive rate of 7.6% per year. As Ghatak, Ghosh, and Kotwal (2014) observe, "It is a period during which growth accelerated, Indians started saving and investing more, the economy opened up, foreign investment came rushing in, poverty declined sharply and building of infrastructure gathered pace . . . [But a] period of fast growth in a poor country can put significant stress on the system which it must cope with. Growth can also unleash powerful aspirations as well as frustrations, and political parties who can tap into these emotions reap the benefits." The same sentiment is echoed by Mishra (2014) in a *Guardian* article just after the elections: "[T]hose made to wait unconscionably long for "trickle-down" —

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people with dramatically raised but mostly unfulfillable aspirations — have become vulnerable to demagogues promising national regeneration."

Following Appadurai (2004) and Ray (1998, 2006), we formulate a theory of socially dependent "aspirations," one that incorporates both the inspiration of higher goals and the potential frustrations that can result. Our starting point is that individual goals are conditioned in fundamental ways by the lives of others. Existing literature views such "reference points" as drawn from the past experience of the individual herself. In contrast, we argue that they are (also) profoundly affected by her *social* environment. This is a view of individual preferences that isn't standard in economic theory. But it should be.

At the same time, while social outcomes affect aspirations, those very aspirations influence — via the aggregation of individual decisions — the overall development of a society. As a result, aspirations and income (and the distribution of income) evolve together. An examination of this relationship is the subject of our paper.

Any such theory must address three issues. First, there is the question of how aspirations are formed. Second, we must describe how individuals react to the aspirations that they do have. Finally, the theory must aggregate individual behavior to derive society-wide outcomes.

We define utilities around "thresholds" and interpret those thresholds as *aspirations*. While guided by theories of reference points — see, e.g., Kahneman and Tversky (1979), Karandikar et al. (1998), and Kőszegi and Rabin (2006) — our point of departure is the dependence of such reference points on both individual wealth *and* the ambient income distribution. In this way we link observed social outcomes to individual behavior.[2]

The crossing of an individual aspiration is "celebrated" by an extra payoff. This "add-on" payoff function is defined on the extent to which outcomes exceed the aspiration, and is exogenous. But the social environment determines the aspiration, and consequently individual incentives to invest and bequeath. This approach allows us to capture both the encouragement and frustration that aspirations can generate, and can be used on its own (i.e., even without any "general equilibrium" considerations) as an aspirations-based theory of poverty traps. We argue that the "best" aspirations are those that lie at a moderate distance from the individual's current economic situation standards, large enough to incentivize but not so large as to induce frustration. Our formulation is in line with evidence from cognitive psychology, sports, education, and lab experiments (see, e.g., Berger and Pope (2011), Goux, Gurgand, and Maurin (2014), Heath, Larrick, and Wu (1999) and Lockwood and Kunda (1997)) that goals that lie ahead — but not too far ahead — provide the best incentives.[3]

While we provide such "partial equilibrium" results, our main contribution is to embed the theory into a simple growth model with evolving income distributions. In equilibrium, the overall income distribution influences individual aspirations, which in turn shape the distribution via

[2]See Macours and Vakis (2009) for evidence of the importance of social interactions in the formation of aspirations.

[3]To cite just one example from social psychology, LeBoeuf and Estes (2004) find that subjects score *lower* on trivia questions when first primed by self-listing the similarities between them and Einstein (what we might interpret as raising their aspirations), relative to when not primed; and they score *higher* when asked to list the differences between them and Einstein (what we interpret as lowering their aspirations) relative to when not primed.

individual choices.[4] We study the properties of equilibrium sequences of income distributions in two environments.

The first is a Solow-like setting in which individual incomes are bounded. In this environment we can define steady state income distributions: those in which the implied set of aspirations feed back to individual decisions, generating the very same distributions with which we started. We argue that such steady state distributions do not, in general, exhibit perfect equality. This is in sharp contrast to the Solow setting where (if all agents are identical and there are no persistent stochastic shocks), one must have convergence: initial differences in wealth die away in the long run. In our setting, excessive compression of the wealth distribution necessarily leads to some individuals accumulating faster than others, and the compression cannot be maintained. When everyone has a common aspiration independent of their wealth, a steady state income distribution of our model is typically bimodal.

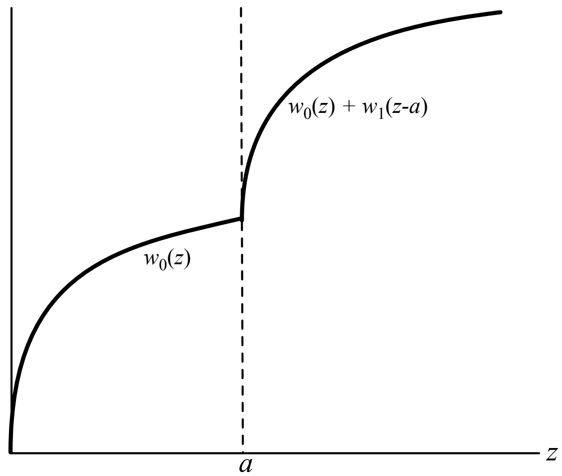
The second environment allows for sustained growth, and assumes constant-elasticity payoff and a linear "A-K" production technology. Now initial conditions determine the asymptotic behavior of the economy. When the initial income distribution is "equal enough," in a sense that we make precise, the economy converges to perfect equality, with all incomes ultimately growing at the same rate. This is akin to the standard convergence predictions of classical growth models. However, when the initial income distribution is unequal, the economy begins to develop clusters, and in lines with the findings of Piketty (2014) and others, inequality must progressively increase; the distribution never settles down, not even to *relative* stationarity. (All formal proofs are in an Appendix.)

These results are in line with a literature that explore various arguments underlying the emergence and persistence of inequality, including nonconvexities (Galor and Zeira (1993), Matsuyama (2004)), occupational choice (Banerjee and Newman (1993), Freeman (1996), Mookherjee and Ray (2003)), institutions (Acemoglu and Robinson (2012), Bowles, Durlauf, and Hoff (2006)), endogenous risk-taking (Becker, Murphy, and Werning (2005), Ray and Robson (2012)), and the "twin-peaks" structure seen in inter-country distributions of per-capita income (Durlauf and Johnson (1995) and Quah (1993, 1996)).

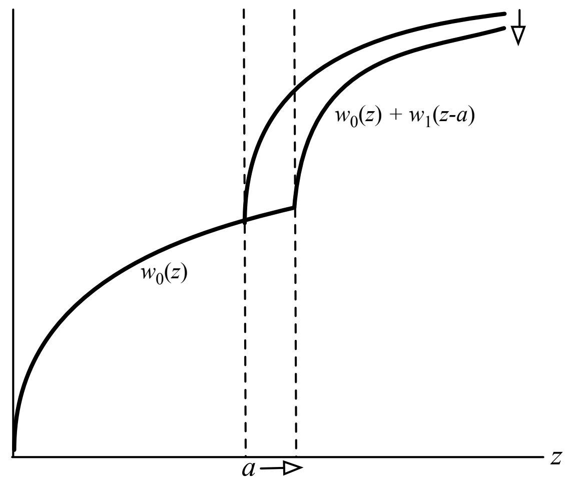
We view this paper as a first step to a theory in which individual goals are socially determined by their economic environment. The explicit description of aspirations as the basic building block through which social influences enter make the model and findings different from a related literature that emphasizes the effect of the ambient distribution on status-seeking (see, e.g., Veblen (1899), Duesenberry (1949), Scitovsky (1976), Frank (1985), Robson (1992), Schor (1992), Clark and Oswald (1996), Corneo and Jeanne (1997), Corneo and Jeanne (1999), Hopkins and Kornienko (2006) and Ray and Robson (2012)).[5] Certainly, our formulation is far from being general and comprehensive. Rather, we seek the essential ingredients of a model that is tractable

[4]This approach develops the ideas laid down in an earlier working paper, Genicot and Ray (2009). In line with that approach, Bogliacino and Ortoleva (2014) also develop a model of socially determined aspirations, while Dalton, Ghosal, and Mani (2014) study a model of internally determined aspirations.

[5]More closely, our approach is related to Karandikar et al. (1998) and Shalev (2000) who endogenize reference points using the realized payoffs of a game. However, the structure we place on aspirations formation as a reference point, and on the "nonlinear" way in which individuals react to the gap between their aspirations and their current standards of living, makes this a distinct exercise, with its own novel distributional and growth implications.



(A) Payoff Function



(B) Change in Aspirations

FIGURE 1. ASPIRATIONS AND PAYOFFS

and useful enough in several applications. The particular application we have emphasized here concerns the evolution of income distributions, retaining the endogenous feedback from distributions to aspirations, and the consequent impact of aspirations on investment and income.

2. ASPIRATIONS, WEALTH DISTRIBUTION AND EQUILIBRIUM

2.1. **An Intertemporal Model With Aspirations.** We study a society populated by a large number of single-parent single-child families. Each person lives for a single period. A sequence of individuals in a family forms a dynasty. A typical member of any generation has a lifetime income (or wealth) y, and allocates y over her lifetime consumption c and investments to affect the wealth of her child z, so as to maximize payoff:

$$u(c) + w_0(z) + w_1(e).$$

There are three terms in this payoff. The first is the utility u from own consumption c. The second and third terms pertain to the utility derived from the child's wealth z. The first of these may be viewed as "intrinsic" parental utility derived from the wealth of the child. Assume that both u and w_0 are increasing, smooth and strictly concave.

The last utility function w_1 represents "milestone utility," the return that parents receive from the excess $e = \max\{z - a, 0\}$ of their child's wealth z over the *aspiration* a of the parent. This aspiration is akin to a reference point, similar to Kahneman and Tversky (1979), Karandikar et al. (1998) and Kőszegi and Rabin (2006), but we will view it as endogenously determined by the parent's socioeconomic environment. We assume that this payoff function is increasing, smooth and strictly concave.

Figure 1, Panel A, depicts the function $w_0(z) + w_1(\max\{z - a, 0\})$. We make five remarks.

First, note that higher aspirations can never increase overall payoffs to the current generation (Panel B). Whether or not a higher a increases the payoffs of the *next* generation depends on how marginal incentives to accumulate are affected, a subject that we take up below. Second, it is possible to write down a variant of this model with *several* aspirational thresholds, such milestones being derived in turn from the overall shape of the distribution. Third, our results are robust to more general forms of the w_1 function that incorporate additional disutility in departing downwards from the aspiration a. What we do require is that such a departure also be concave, so that the extent of frustration in falling below the aspiration exhibits diminishing returns. It is central to our analysis that the aggregated function $w_0 + w_1$ display sufficient nonconcavity around the aspirational threshold. Fourth, our results are made even stronger if the crossing of the aspiration threshold engenders a discontinuous jump in utility; i.e., if $w_1(0) > 0$.

And finally, the intrinsic term w_0 is defined on the wealth of the child, but it is not a value function in the sense of dynamic programming. Writing such a version of this problem is possible but complicated, as it requires parents to forecast the endogenously determined aspirations of their children, grandchildren, and so forth.

2.2. **The Formation of Aspirations.** Two alternative approaches, by no means mutually exclusive, connect aspirations to economic outcomes and so bring the theory full circle. One possibility is to take an entirely *private* viewpoint: one's personal experiences determine future goals, so that each individual can be analyzed as as a self-contained unit. This is the approach taken in Karandikar et al. (1998) and Kőszegi and Rabin (2006) when determining reference points; see also Alonso-Carrera, Caball, and Raurich (2007), Carroll and Weil (1994) and Croix and Michel (2001). In this literature, the loop that runs from reference points to behavior and back to reference points is entirely internal to the individual.

In contrast, economic models of status (see the many references in the Introduction) achieve closure by using *social* outcomes external to the individual. A broad array of possibilities is captured under the specification

$$a = \Psi(y, F), \tag{1}$$

where y is lifetime individual wealth (as above) and F is the society-wide distribution of lifetime incomes in the current generation.[6]

Notice that aspirations are allowed to depend both on personal and social circumstances, though everyone is presumed to have the same aspirations formation *function*. This allows for a substantial amount of effective heterogeneity, induced endogenously via varying wealths. We will impose the following four conditions on Ψ at different points in the paper:

[6] It is, of course, also possible to adopt a specification in which the *anticipated* distribution of wealth over future generations drives aspirations. A previous version of the paper, see Genicot and Ray (2009), discusses and compares the two approaches. We are comfortable with either model, but adopt the current approach for two reasons: (a) it uses the perhaps more satisfying formulation that goals are derived from an *actual* situation rather than an anticipated state of affairs which may or may not come to pass, and (b) the resulting structure is fully recursive and far more friendly to numerical computation.

First, we assume throughout (without explicit mention) that Ψ is continuous and nondecreasing in (y, F).[7] Next, say that Ψ is *range-bound* if $\min F \leq \Psi(y, F) \leq \max F$ for all F and $y \in$ Supp F.[8] The justification for imposing this restriction is that aspirations represent *social* achievements; they are stepping stones that are measured relative to what one's own compatriots are experiencing, and so individuals do not use aspirations that are located beyond the full income range of whatever they see around them. Third, aspirations are *scale-free* if $\Psi(\lambda y, \lambda F) = \lambda \Psi(y, F)$ for all $\lambda > 0$, where λF stands for the distribution obtained when all incomes in F are scaled by λ. Finally, aspirations are *socially sensitive* if a scaling-up of incomes everywhere increases individual aspirations for fixed individual income; i.e., $\Psi(y, \lambda F) > \Psi(y, F)$ for all y. This is akin to saying that if someone's income stays the same while everyone else's incomes increase, that person's aspirations should go up (and he or she would feel worse). We return to a discussion of this assumption in Section 4.4.

As an example, suppose that aspirations are given by some weighted average of one's own income and the overall mean of the distribution, where these weights are unchanging in income. As long the weight on overall mean is strictly positive, then all the conditions introduced above are satisfied.

2.3. **Dynastic Equilibrium With Aspirations.** To describe equilibria, embed this model of aspirations formation into a standard growth model. An individual with wealth w_t and aspirations a_t divides her wealth between consumption c_t and a bequest for the future, k_t:

$$y_t = c_t + k_t.$$

That bequest gives rise to fresh wealth for the next generation:

$$y_{t+1} = f(k_t),$$

where f is a smooth, increasing, concave function. An individual *policy* ϕ maps wealth y_t and aspirations a_t to wealth $z_t = y_{t+1}$ for the next generation.

An *equilibrium* from some initial distribution F_0 is a sequence of income distributions $\{F_t\}$ and a policy ϕ such that

(i) At every t, aspirations are given by $a_t = \Psi(y_t, F_t)$ for each individual with wealth y_t, and $z_t = \phi(y_t, a_t)$ maximizes

$$u\left(y_t - k(z)\right) + w_0\left(z\right) + w_1\left(\max\{z - a_t, 0\}\right) \tag{2}$$

over $z \in [0, f(y)]$, where $k(z) \equiv f^{-1}(z)$.

(ii) F_{t+1} is generated from F_t and the policy ϕ; that is, for each $z \geq 0$,

$$F_{t+1}(z) = \text{Prob}_t\{y|\phi(y, \Psi(y, F_t)) \leq z\},$$

where Prob_t is the probability measure induced by the distribution function F_t.

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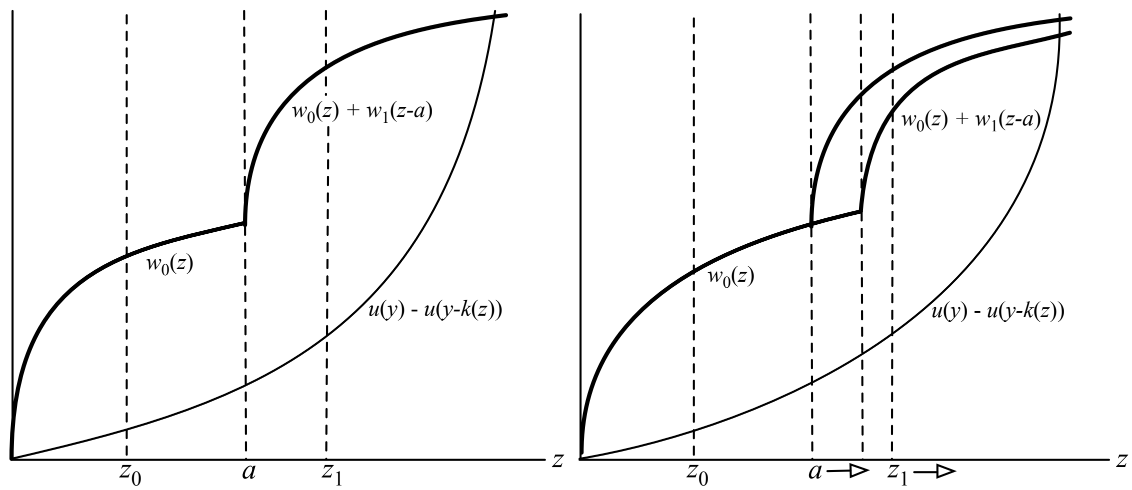
[7] Continuity in F is defined with respect to the topology of weak convergence on distributions. By "nondecreasing in F," we mean that if F' dominates F in the sense of first-order stochastic dominance, then $\Psi(y, F') \geq \Psi(y, F)$.

[8] The terms $\min F$ and $\max F$ refer to the minimum and maximum values in the support of the distribution F.



(A) Costs and Benefits

(B) Change in a

FIGURE 2. THE CHOICE OF FUTURE WEALTH.

Note that *given the aspirations*, there is no particular need for the policy function to be time-dependent; the resulting maximization problem (2) is entirely stationary.

Proposition 1. *An equilibrium exists.*

The proof of this proposition is a simple recursive exercise, starting from any initial F_0.

3. THE EFFECT OF ASPIRATIONS AND WEALTH ON INVESTMENT

3.1. **Benchmark with No Aspirations.** In an artificial benchmark model without any aspirations at all, an individual would choose z to maximize

$$u(y - k(z)) + w_0(z)$$

where recall that $k(z) = f^{-1}(z)$. To avoid completely uninteresting cases where income is zero, assume that the system "pushes away" from 0 at low positive values of income; that is,

$$-\frac{u'(y - k(y))}{f'(k(y))} + w_0'(y) > 0 \tag{3}$$

for $y > 0$ and small enough. We maintain this condition throughout.

3.2. **Aspirations and Choices.** Now return to our model. Figure 2 shows us how to graphically think about the maximization problem induced by expression (2). The horizontal axis plots the choice of future wealth z, while the vertical axis records various benefits and costs. The benefit that accrues from next generation's wealth is given by

$$w_0(z) + w_1(e_1),$$

where $e_1 = \max\{z - a, 0\}$ is the excess (if any) of wealth over the threshold a. The cost is the sacrifice of current utility, which we can write as $u(y) - u(y - k(z))$. Panel A of Figure 2 plots both these functions. The "cost function" has a standard shape: it is the convex lower curve. Given income and aspirations, our maximization problem seeks a continuation income z that produces the largest vertical distance between these two curves.

By the concavity of benefits to the left and right of a, there can be at most one "local" solution on either side of a. Finding an optimal solution involves comparing these two local solutions, that is, solving at most two interior first-order conditions

(4) $$w_0'(z_0) = u'(y - k(z_0)) / f'(k(z_0))$$

and

(5) $$w_0'(z_1) + w_1'(z_1 - a) = u'(y - k(z_1)) / f'(k(z_1)),$$

the former applicable if $z_0 < a$ and the latter if $z_1 > a$, and picking the one that yields the higher payoff.[9]

Say that the aspiration a is *satisfied* if the chosen optimal solution exceeds a, and *frustrated* if it falls short of a. (The slight ambiguity in this definition will be excused as the optimal choice of z will be generically be unique, with multiple solutions possible only for knife-edge values of (y, a). See Proposition 2 below.)

3.3. **Changing Aspirations.** When is an aspiration threshold satisfied, and when is it frustrated? We can examine this question by varying aspirations for a fixed level of income, or by varying income for some fixed aspiration. In this subsection, we consider an exogenous change in that aspiration for some individual with given income. Such changes don't just constitute an abstract exercise. For instance, the rise of mass media in developing countries (such as television, advertising or the internet) will bring particular socioeconomic groups into focus, thus affecting aspirations.[10] In addition, a change in aspirations can be fueled by growth or decay in the ambient income distribution.

When the aspiration is close to zero, the optimal solution must strictly exceeds the aspiration, and so aspirations are satisfied. As long as aspirations remain in the "satisfaction zone," an increase in aspirations incentivizes growth. As the threshold continues to rise, there comes an aspiration level a^* illustrated in Panel A of Figure 3 when the solution makes a sudden switch from satisfaction to frustration: this switch will arrive with a discontinuous fall in investment, as is evident by consideration of the two first-order conditions (4) and (5). Once in the "frustration zone," investment becomes insensitive to further increases in aspirations. Proposition 2 formalizes this discussion:

[9]More precisely, write down the first-order condition (4). By (3), an interior solution always exists. If $z_0 \geq a$, then we know that the first-order condition (5) must be the relevant one, and the solution to the problem is given by z_1. If $z_0 < a$, then check to see if (5) is satisfied for any $z_1 > a$. If it isn't, then z_0 is the solution to our problem. If a solution to (5) does exist for $z_1 > a$, then choose whichever solution yields the higher payoff. Ties can be broken arbitrarily.

[10]See Jensen and Oster (2009) and Ferrara, Chong, and Duryea (2012) for evidence on how the introduction of cable television can expose people to very different lifestyles, thereby affecting their aspirations and fertility preferences.

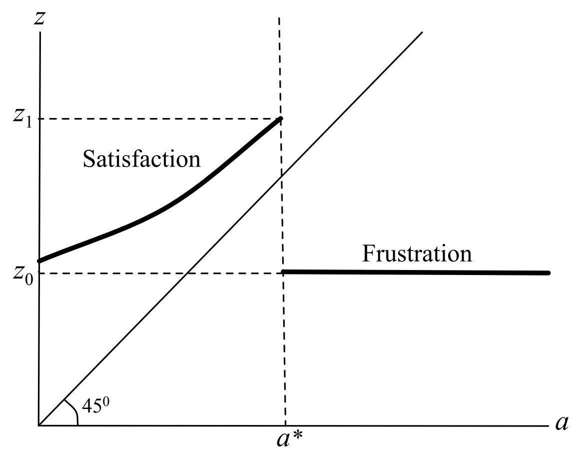


FIGURE 3. SATISFACTION AND FRUSTRATION AS ASPIRATIONS CHANGE.

Proposition 2. *For any given initial wealth, there is a unique threshold value of aspirations below which aspirations are satisfied, and above which they are frustrated. As long as aspirations are satisfied, chosen wealth grows with aspirations. Once aspirations are frustrated, chosen wealth becomes insensitive to aspirations.*

Our observations on frustration are consistent with the arguments of Appadurai (2004) and Ray (1998, 2006), and with a recent literature that argues that lowering the aspirations of low income students to more reachable levels reduces the likelihood of dropping out of school in the US (Kearney and Levine (2014)) and in France (Goux, Gurgand, and Maurin (2014)). Or for instance, Indian liberalization in the 1990s and its subsequent growth story combined with an explosion of social media, from television to the internet, has undoubtedly raised aspirations everywhere. The rise of an economically powerful urban middle class is certainly consistent with a story of burgeoning aspirations with salubrious effects on investment. But there is a second story to be told, in which large sections of the population are effectively delinked from the growth process (see Banerjee and Piketty (2005), Deaton and Drèze (2002) and Drèze and Sen (2013)) and, along with the success stories that foreign investors so like to hear, there is a subtext of apathy and despair, violence and conflict. Whether the potential for frustration caused by rising aspirations plays a central role in this story deserves more investigation and research. But the observations are *prima facie* consistent with such a story.

3.4. **Changing Wealth.** The second piece of our exercise concerns changes in individual wealth. To be sure, that will generally cause a change individual aspirations as well, the precise nature of the change depending on the ambient distribution of income in society. In what follows we study how the optimally chosen *growth rate* of wealth varies along the cross-section of "starting wealths" recognizing that aspirations may change with wealth. To do so, we introduce an important special case of our model that is particularly conducive to the study of endogenous growth. Call it the *constant elasticity growth model*. It has the following components. First, we impose an "A-K" setting in which the production function is linear:

$$f(k) = \rho k \tag{6}$$

where $\rho > 1$ is some constant return on capital holdings. This formulation accommodates exogenous labor income in every period. We can also include a credit limit on borrowing based on future labor income; it will make no difference to the results.[11]

Next, we assume that utilities are constant-elasticity, with the same elasticity for each utility indicator:

$$u(c) = c^{1-\sigma}, w_0(z) = \delta z^{1-\sigma}, \text{ and } w_1(e_i) = \delta \pi e^{1-\sigma} \tag{7}$$

where $\sigma \in (0,1)$, $\delta > 0$ is a measure of discounting, $\pi > 0$ is a measure of the additional value of crossing the aspiration, and e is the excess of z over aspirations.[12]

The expositional advantage of the constant elasticity growth model is that, in the absence of an aspirations effect, bequests are proportional to wealth and growth rates are constant across the cross-section of current wealths. We can therefore be sure that any cross-sectional variation in growth rates stems entirely from aspirations alone. We describe the *growth incidence curve*, a relationship that links baseline income to subsequent rates of growth.

To this end, note that an individual with starting wealth y and aspirations a will choose continuation wealth z to maximize

$$\left(y - \frac{z}{\rho}\right)^{1-\sigma} + \delta\left[z^{1-\sigma} + \pi\left(\max\{z-a, 0\}\right)^{1-\sigma}\right]. \tag{8}$$

Let $r \equiv y/a$ denote the *aspirations ratio*: the ratio of the baseline wealth to aspirations. The maximization in (8) is equivalent to choosing a growth factor $g \equiv z/y$ that maximizes

$$\left(1 - \frac{g}{\rho}\right)^{1-\sigma} + \delta\left[g^{1-\sigma} + \pi\left(\max\{g - \frac{1}{r}, 0\}\right)^{1-\sigma}\right]. \tag{9}$$

We solve this problem just as in the general case. First write down the first-order condition under the assumption that aspirations are met; that is, $g \geq \frac{1}{r}$. The corresponding growth factor $g(r) \equiv z/y$ is given by the solution to

$$\left(1 - \frac{g(r)}{\rho}\right)^{-\sigma} = \delta\rho\left[g(r)^{-\sigma} + \pi\left(g(r) - \frac{1}{r}\right)^{-\sigma}\right]. \tag{10}$$

[11]As in Bernheim, Ray, and Yeltekin (2015), if each generation earns a constant labor income ℓ in addition to receiving bequests, then $y_t = f_t + [\rho\ell/(\rho-1)]$, where f_t is financial wealth. If she can borrow some fraction $(1-\lambda)$ of future income, then that translate into a lower bound on total wealth y, given by $B = \lambda\rho\ell/(\rho-1)$.

[12]Given constant elasticity, the use of a common elasticity term σ for the utility and aspirational components is all but unavoidable (once we incorporate the notion that aspirations move in tandem with income). To see why, imagine scaling up aspirations and income together, which is what will happen in the sequel when incomes are growing and aspirations are growing along with incomes. If the elasticities are not the same, then at least one of these three terms will either become relatively insignificant or unboundedly dominant. To retain the relative importance of both intrinsic consumption and aspirations, we use the same elasticity for each of these functions.

Note that there is a unique solution $g(r)$ to this equation in the region $(\frac{1}{r}, \infty)$ as long as this region is "reachable," which it will be provided $\rho > \frac{1}{r}$.[13] Moreover, it is easy to check that $g(r)$ is strictly decreasing in r.

The other option is for the individual to entirely ignore aspirations, which yields (via the usual first order condition), the growth factor $\underline{g}$ that solves:

$$\left(1 - \frac{\underline{g}}{\rho}\right)^{-\sigma} = \delta\rho\underline{g}^{-\sigma} \tag{11}$$

or

$$\underline{g} = \frac{\rho}{1 + \delta^{-\frac{1}{\sigma}}\rho^{-\frac{1-\sigma}{\sigma}}}. \tag{12}$$

If $\underline{g}$ turns out to exceed $1/r$, then it is obvious that the $g(r)$-solution is optimal at y (because the latter "includes" the aspirational payoffs while the construction of $\underline{g}$ does not). On the other hand, if $\underline{g} < 1/r$, then the individual must compare payoffs from the two alternative choices given by (10) and (11).[14]

Proposition 3. *In the constant elasticity growth model, there is a unique threshold $r^* < 1$ such that for all wealth-aspiration pairs (y, a) with $r \equiv y/a < r^*$, continuation wealth grows by the factor $\underline{g}$, and for all (y, a) with $r \equiv y/a > r^*$, continuation wealth grows by the factor $g(r)$. This value $g(r)$ declines in r, but is always strictly larger and bounded away from $\underline{g}$ on (r^*, ∞).*

The proposition states that for low levels of the aspirations ratio, growth rates are at their "frustration level" $\underline{g}$. Depending on the parameters, this rate may imply growth or decay. As the ratio climbs, there is a threshold r^* at which the "upper solution" to the first order condition (10) dominates the "lower solution" to the first order condition (11), and the growth rate jumps upward. Assumption 3 implies that growth rates are positive and therefore that this jump threshold lies below 1.

Thereafter, as growth rates fall again as the aspirations ratio continues to rise, but because of the additional marginal payoff bestowed by aspirations utility, never come down to the original "frustration rate" $\underline{g}$.

Notice that this proposition says nothing about initial *incomes*, but maps initial *ratios* of income and aspirations to subsequent rates of growth. The proposition does translate into a growth incidence curve once we can connect initial incomes to aspirations ratios. To this end, note that for any distribution F and income y, the aspirations ratio is given by

$$r(y, F) \equiv y/\Psi(y, F).$$

We can provide conditions under which aspirations ratios are increasing along the cross-section of wealth.

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[13]If $\rho > \frac{1}{r}$, then starting at income y it will be possible to produce more than a: $\rho y > a$.

[14]That is, she checks if $\left(1 - \frac{g(r)}{\rho}\right)^{1-\sigma} + \delta\left[g(r)^{1-\sigma} + \pi\left(g(r) - \frac{1}{r}\right)^{1-\sigma}\right] > \left(1 - \frac{\underline{g}}{\rho}\right)^{1-\sigma} + \delta\underline{g}^{1-\sigma}$, chooses $g(r)$ if this inequality holds, and $\underline{g}$ otherwise.

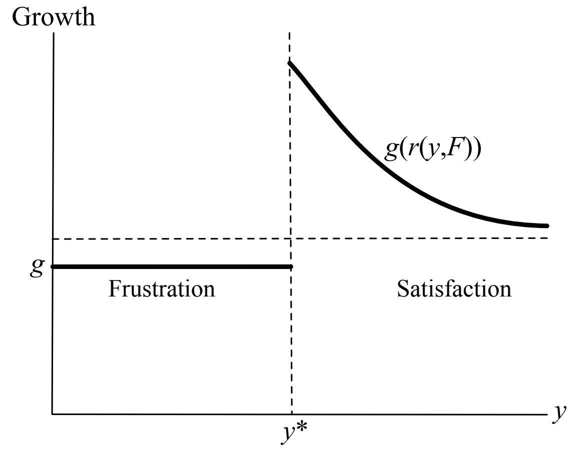


FIGURE 4. GROWTH RATE AS A FUNCTION OF INITIAL WEALTH.

Lemma 1. *If aspirations are scale-free and socially sensitive, then the aspirations ratio $r(y, F)$ is strictly increasing in y, for given F.*

Lemma 1 combines immediately with Proposition 3 to yield

Proposition 4. *In the constant elasticity growth model, assume that aspirations are scale-free and socially sensitive. Fix income distribution F and vary initial income y. Then there is a threshold y^* such that the growth rate is $\underline{g}$ as long as $y < y^*$, jumps up at y^* and thereafter declines in y, but remains strictly larger and bounded away from $\underline{g}$.*

Figure 4 illustrates Proposition 4. Roughly speaking, the growth incidence curve is inverted-U shaped.

4. THE JOINT EVOLUTION OF ASPIRATIONS AND INCOMES

In the previous section, we emphasized some partial effects of aspirations and wealth on the subsequent growth of incomes. Because aspirations and incomes evolve jointly, these effects intertwine, depending on the precise manner in which aspirations are formed. Section 2.3 formally defines an equilibrium sequence of income distributions $\{F_t\}$. There are natural questions that one can ask of such a formulation. For instance:

(i) Does the general equilibrium of aspirations and income foster persistent inequality in "steady state"?

(ii) What is the relationship between initial inequality and subsequent growth?

(iii) How does inequality evolve along a growth path?

The first of these questions is naturally suited to a setting in which incomes lie in some bounded interval (such as the Solow model), while the remaining questions are germane to a setting in which sustained growth is possible.

4.1. **Bounded Incomes and Stationary States.** Consider distributions on strictly positive values of wealth F^* such that each dynasty replicates its starting wealth generation after generation. Call these *stationary states*. A somewhat weaker but more esoteric definition is that of a *steady state*: a distribution that replicates itself period after period. The two are separated by the possibility that in the latter, dynasties might "cross paths": the fact that aspirations move endogenously can destroy the single-crossing property and allow for endogenous mobility within a steady state, whereby individuals with failed aspirations can cross over and fall below lower-wealth individuals with satisfied aspirations. Modulo this property, the two definitions are the same.

A natural setting for steady states is one in which all wealths lie in some bounded set, as in the Solow model (after normalizing for technical progress and population growth). It will be implied by the following sort of restriction: $f(x) < x$ for all x large enough.

Proposition 5. *If aspirations are range-bound, no steady state can involve perfect equality of wealth.*

This proposition comes from the convexification of utility caused by the presence of aspirations. Range-boundedness implies that when incomes converge to each other, aspirations must lie in this narrow range as well. However, when incomes are very close to current aspirations, the marginal utility of accumulation is high in one direction and low in the other, and the system must push away from this neighborhood. Whether it pushes upwards or downwards will depend, as before, on a comparison with the two locally optimal choices on either side of the current aspiration level. But the essential point is that the system *cannot* stay where it is. Therefore, the only way to have a steady state is to have a multitude of incomes populating that steady state, even without any fundamental uncertainty. The local convexity of aspirations-based utility (around the aspiration level) precludes convergence.

How seriously we take this result depends on one's intuition about marginal utility as one departs from incomes close to aspirations. Aspiration-fulfillment does imply that an important goal has just been reached, and to the extent that there is some fundamental component of satisfaction that depends on the crossing of that goal, and an important notion of failure on not reaching it, local convexification is not an unrealistic property, and our model has that property.[15] An upward payoff discontinuity at the aspiration threshold would, of course, additionally strengthen the result.

There is a second assumption that drives our result on steady state inequality. This is the condition that aspirations are range-bound. As already discussed, aspirations are social milestones.

[15]While such convexification could be outweighed by some other form of concavity in the system, such as the curvature of the production function, our model rules out this possibility by juxtaposing a fresh source of utility (the function w_1) as the aspiration level is crossed on top of the existing utility from progeny income (the function w_0), thereby creating a kink that "dominates" any degree of (smooth) curvature in f.

They must lie in the realm of what one's compatriots are experiencing. Range-boundedness implies that individuals do not use aspirations that are located beyond the full income range of whatever they see around them.

Proposition 5 is related to different aspects of the literature on evolving income distributions. The closest relationship is to endogenous inequality, in which high levels of equality are destabilized by forces that tend to move the system away from global clustering. In Freeman (1996) and Mookherjee and Ray (2003), this happens because of imperfect substitutes among factors of productions, so that a variety of occupations with different training costs and returns *must* be populated in equilibrium. Together with imperfect capital markets, this implies that in steady state, there must be persistent inequality, even in the absence of any stochastic shocks. In a different context, Becker, Murphy, and Werning (2005) and Ray and Robson (2012) argue that endogenous risk-taking can also serve to disrupt equality, as relative status-seeking effectively "convexifies" the utility function at high levels of clustering.

4.2. **Bimodality.** Under additional conditions, a stationary distribution of income must not only be unequal, but exhibit bimodality.[16] To examine this phenomenon, and to avoid complications that have nothing to do with aspirations, recall the benchmark model *without* aspirations from Section 3.1. A *benchmark stationary income* y^* is a positive income level for which the corresponding choice of continuation income z *equals* y^*. Given (3), a benchmark stationary income must be interior, and is characterized by the condition

$$d(y) \equiv -\frac{u'(y-k(y))}{f'(k(y))} + w_0'(y) = 0. \tag{13}$$

In what follows, we will assume

[D] $d(y)$ is decreasing in y.

So in particular, the benchmark stationary income is unique.[17] This purges our model of possible inequalities that might arise from a "super-normal" response of child wealth to parental wealth (in a standard setting without changed aspirations).

Proposition 6. *Assume that aspirations are range-bound, scale-free and socially sensitive, and impose Condition D. Then every stationary state is concentrated on just two positive values of incomes.*

Proposition 6, stark as it is, is not meant to be taken literally. Convergence to *degenerate* poles is an artifact of the assumptions. (That would be akin to stating that the Solow model predicts a *single* income level in steady state.) When there are stochastic shocks, the distribution will always be dispersed, as in the extension of the standard growth model by Brock and Mirman (1972) and others. We could easily introduce such shocks into the model at hand. The following example does just that.

[16]We are not aware of a similar result for steady state distributions.

[17]It is easy enough to write down specific functional forms that satisfy this requirement. For instance, if $f(k) = Ak^{\alpha}$, then [D] holds whenever $-u''(c)c/u'(c) \leq (1-\alpha)$ for all consumption levels c.

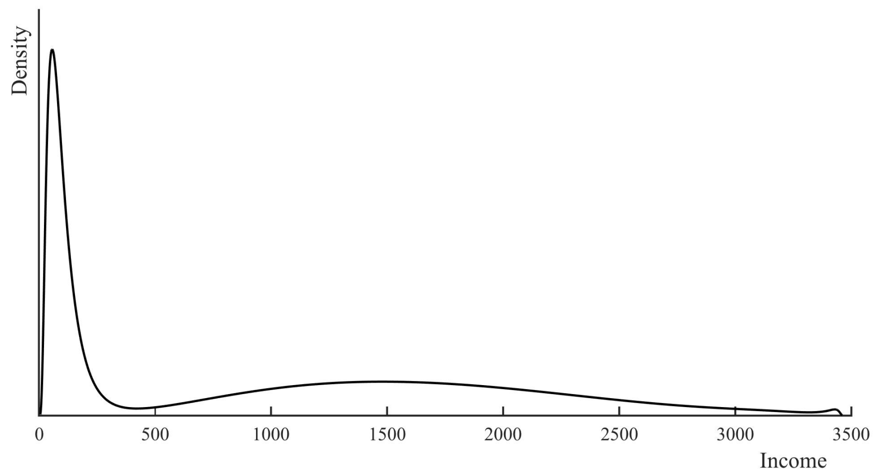


FIGURE 5. BIMODALITY IN A STATIONARY STATE.

Example 1. Utilities are of the constant-elasticity form introduced in (7). We set $\sigma = 0.8$, $\delta = 0.8$ and $\pi = 1$. In order to get non-degenerate (and therefore more realistic) distributions of income, we introduce some noise in the production function. We take the production function to be $f(k, \theta) = \theta(A/\beta)k^{\beta}$, where $\beta = 0.8$, $A = 4$ and θ is a stochastic shock with mean 1.[18] We set aspirations to the average of one's own income and the mean income, begin with an initial distribution of income that is uniform over a population of 300 individuals, and iterate the distribution over time. The simulated distributions converge rapidly to a bimodal distribution shown in Figure 5 where the only mobility is due to the noise in the production function.[19]

There is evidence of clustering in the income distribution of various countries, including the United States (see Pittau and Zelli (2004), i Martin (2006) and Zhu (2005)), and especially Durlauf and Johnson (1995), and Quah (1993, 1996).[20] These authors make a strong case for local clustering in the world income distribution and argue that convergence is a local phenomenon "within the cluster" but not globally. Quah refers to these local clusters as "convergence clubs." Durlauf and Quah (1999) summarize by writing that there is an "increase in overall spread together with [a] reduction in intra-distributional inequalities by an emergence of distinct peaks in the distribution."

We learned from Proposition 6 that a stationary state distribution F^* takes the form of a two-point distribution (y_ℓ, y_h, p), where $y_\ell < y_h$, with p the population weight on y_ℓ. For each group $i = \ell, h$, a_i is then given by

$$a_i = \Psi(y_i, F^*). \tag{14}$$

[18]Specifically, we suppose that θ follows a lognormal distribution. The qualitative results do not depend on the magnitude of the noise term, though in general, the degree of clustering must rise as the variance of the shock falls.

[19]In the figures, we smoothed the simulated distribution using the density estimator "ksdensity" for Matlab. In the absence of noise, the distribution concentrates on two levels on income: 95 and $1,779$.

[20]See also Henderson, Parmeter, and Russell (2008), Canova (2004) and Pittau, Zelli, and Johnson (2010).

We know, moreover, that a_ℓ *must* be a failed aspiration for y_ℓ, so that the steady-state income level y_ℓ is fully pinned down by

$$d(y_\ell) = 0, \tag{15}$$

for which the solution is unique, by Condition D. On the other hand, a_h must be a satisfied aspiration for y_h, so that y_h is determined by

$$d(y_h) + w_1'(y_h - \Psi(y_h, F^*)) = 0. \tag{16}$$

(We note in passing that Proposition 6 is proved by showing that there cannot be more than one solution to (16), for any F^*.)

These are four equations for five unknowns $(y_\ell, y_h, p, a_\ell, a_h)$, but in part the extra degree of freedom will be used up in guaranteeing that we can find configurations that are compatible with the failure of aspirations at y_ℓ and the satisfaction of aspirations at y_h.[21]

Within the range of stationary states, there is a close relationship between the proportion of low-income earners, and the income gap between high and low incomes. As one moves from a stationary state with a high proportion of low-income earners to another with a smaller proportion, the income gap widens. A similar feature of widening inequality with growth will reappear more explicitly when we consider dynamic paths with sustained growth. This is illustrated in the following example.

Example 2. In this example, we assume the same preference structure and production function as in Example 1 but without noise. We set $\sigma = 0.8$, $\delta = 0.8$ and $\pi = 1$ for the preferences, and $A = 4$ and $\beta = 0.55$ for the production function. As before, aspirations are set at the average of one's own income and the mean income. The lowest level of income in steady state — determined by (15) — is $y_\ell = 21$. There is a range of steady states characterized by combinations of high income values (y_h) and low income population proportions (p). As in the discussion above, a smaller "poor population" is also associated with a higher income for the rich and greater inequality. Notice that only values of p between $p_{\text{min}} = 0.03$ and $p_{\text{max}} = 0.25$ are possible in steady state. Figure 6 shows the high income y_h and the level of aspiration as a function of p in steady state. The high incomes adjust accordingly steadily declining from 79 to 66.

4.3. **Growth and Inequality.** We now turn to a different scenario which accommodates endogenous growth. To this end, we return to the constant elasticity growth model introduced in Section 3.4. Recall that in that model, all utility indicators are constant-elasticity with the same elasticity, and the production function is linear.

Our starting point is some initial distribution of wealth; call it F_0. Initial aspirations are given by the mapping $a_0(y) = \Psi(y, F_0)$ for every income y in the support of F_0. An individual with

[21]It should be noted that a stationary state may not exist. After all, a_ℓ must be bounded below by $\underline{a}$, which is the lowest aspiration for which an individual at y_ℓ is just indifferent between her lower choice $z = y_\ell$ and some upper choice $z' > \underline{a}$. In turn, that places a lower bound on the size of y_h. At that value, it is possible that the production function has marginal product low enough so that that level of income is not worth maintaining for any aspiration a_h. Faced with this possible nonexistence, one must then retreat to the weaker definition of a steady state, but we do not claim that Proposition 6 holds for all steady states.

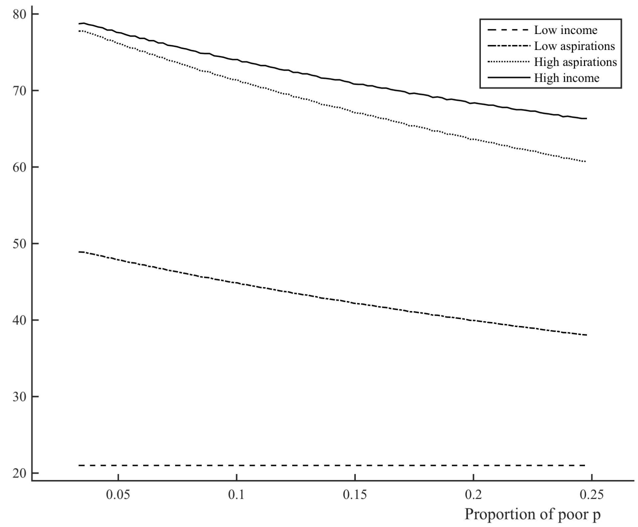


FIGURE 6. INEQUALITY AND COMMONLY HELD ASPIRATIONS.

income y will choose continuation wealth z to maximize

$$\left(y - \frac{z}{\rho}\right)^{1-\sigma} + \delta\left[z^{1-\sigma} + \pi\left(\max\{z - a_0(y), 0\}\right)^{1-\sigma}\right]$$

Recalling the analysis of Section 3.4, two choices need to be compared. The higher of the choices involves the growth rate that solves equation (10). This solution, which we denote by $g(r)$, depends on baseline wealth y but only via the aspirations ratio $r = y/a_0(y)$; see Section 3.4. The lower of the two choices involves a growth rate of $\underline{g}$, which solves (12): this choice is entirely independent of y. We adopt the innocuous convention that if an individual is indifferent between the higher and lower growth rates, she chooses the higher rate.[22]

Proposition 7. *Consider the constant-elasticity growth model. Assume that aspirations are range-bound, scale-free and socially sensitive. Let F_0 be some initial distribution of wealth with compact support. Then there are just two possibilities:*

1. Convergence To Perfect Equality*: All wealths grow asymptotically at the rate $g(1) - 1$, and normalized incomes $y_t/g(1)^t$ converge to a single point independent of $y_0 \in$* Supp F_0.

Or there is

2. Persistent Divergence: *F_t "separates" into two components. There is a critical income level y^* in the interior of the support of F_0 such that all incomes below y^* change thereafter by the growth factor $\underline{g}$. All incomes initially above y^* grow by some asymptotic factor $\bar{g} > \underline{g}$, with $\bar{g} - 1 > 0$. Moreover, there is normalized convergence of these incomes: $y_t/\bar{g}^t$ converges to the*

[22]Any other convention works just as well, and so does no convention at all, but the analysis will then need to keep track of potential discontinuities that might merge at the knife-edge threshold r^*, so we simplify the exposition by adopting a convention.

same limit irrespective of y_0, as long as y_0 exceeds y^*. Overall, relative inequality never settles down: despite the within-group convergence, it increases without bound.

Growth Comparison*: With divergence, the asymptotic growth rate of every individual is no higher than the common asymptotic growth rate under perfect equality, and strictly lower for a positive measure of individuals.*

Proposition 7 significantly narrows the way in which the dynamics of an income distribution can evolve. There are only two possibilities.

In the first of these, every initial income level has satisfied aspirations. That means that the initial distribution has a high level of equality to begin with, so that even the lowest income level is not frustrated by the aspirations generated under F_0. That may be a tall order, but if it is met, then indeed all incomes converge to perfect equality with sustained growth. Thus the basin of attraction for an equal steady state with growth *is a relatively equal society to begin with*.

If that condition is not met, then the second possibility arises. Incomes at the lower end fall short of aspirations, the economy turns bimodal and inequality increases. Moreover, that inequality never stops increasing, *even in relative terms*, with the income ratio between the haves and the have-nots steadily rising. However, there is growth-rate convergence among the "haves," and there may even be level convergence (after normalizing by the growth rate).

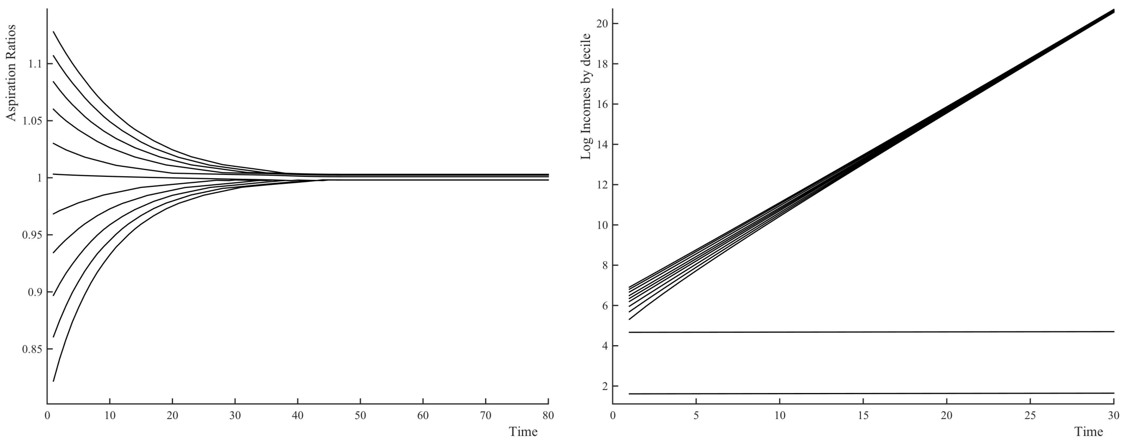
There is a third possibility: that *every* individual has frustrated aspirations under F_0. Then everyone chooses the growth rate $\underline{g}$ in (12), and next period's wealth distribution will just be a proportional scaling of all incomes in F_0 by the factor $\underline{g}$. This case is ruled out under the restriction (3) as we show in the Appendix. (Even if we did not assume (3), so that this case becomes logically possible, it would be entirely uninteresting because it involves universally frustrated aspirations and perpetual decay.)

Proposition 7 is illustrated by the following example.

Example 3. We use the same preferences as in Example 1 with aspirations given by an average of individual income and mean income, and a linear production function without noise with coefficient $\rho = 2.1$. We consider 1000 individuals and the evolution of their income over time. Figure 7 plots the evolution of the log income of the poorest individual and every decile thereafter (the individual at the 10th percentile, 20th percentile, etc) up to the richest individual in the economy. Since there is no relative mobility, higher curves represent richer individuals.

In Panel A we start with a uniform distribution ranging from 350 to 650, and observe convergence to equality with all incomes and aspirations growing at the factor $g(1) = 2.04$. We display the panel by normalizing trajectories by this limit growth rate, so that convergence can be clearly seen.

In Panel B we start with a uniform distribution ranging from 5 to 1000 and end up with bipolar divergence. Individuals with less than 157 choose a growth rate of 0.001 ($\underline{g} = 1.001$); their aspiration ratios progressively decline over time. In contrast, individuals with initial income that exceed 157 choose a higher growth rate. In the limit, the growth-normalized incomes of all these



(A) Equal Initial Distribution (growth-normalized) (B) Unequal Initial Distribution (log-linearized)

FIGURE 7. AN ILLUSTRATION OF PROPOSITION 7.

individuals converge, and the common limit growth factor is $\bar{g} = 1.63$. Notice that this growth rate is less than the growth ultimately experienced by all in the more equal society of Panel A.

In addition to a more equal initial distribution, we may be interested in other factors that "increase the chances" of convergence to equality. One way to formalize this is to say that some parametric change makes convergence to equality *more likely* if for any initial distribution F_0 with convergence to perfect equality before the change, that convergence is unaffected, and for some distributions F_0 with bipolar divergence before the change, convergence to perfect equality occurs after the change. Observation 1 describes the effect of various parameters on the likelihood of convergence to a stationary state with perfect equality. When looking at the effect of social sensitivity, we consider aspirations to be a weighted average on one's own income y and a common term $\psi(F)$ in the range of the distribution:

$$\Psi(y, F) = \gamma y + (1 - \gamma)\psi(F) \text{ for } \gamma \in [0, 1], \tag{17}$$

so that a higher γ means less socially sensitive aspirations.

Observation 1. *Convergence to perfect equality is more likely for (i) higher rates of returns (ρ), (ii) lower aspirations (that is, a decrease in $\Psi(y, F)$ for all (y, F)) and (iii) less socially sensitive aspirations (higher γ).*

Proposition 3 showed that whether an individual with income y is frustrated or satisfied at time 0 — and therefore at any future date, as implied by Proposition 7 — depends on whether that individual's aspirations ratio $r_0(y)$ is below or above the threshold r^* identified in the statement of Proposition 3. Scenario 1 of convergence to perfect inequality obtains when $r_0(y) \geq r^*$ for all incomes y in the initial distribution F_0.

It is obvious from the maximization problem in (8) that a higher rate of return ρ reduces the threshold r^*, thereby reducing frustration and so increasing the likelihood of perfect equality.

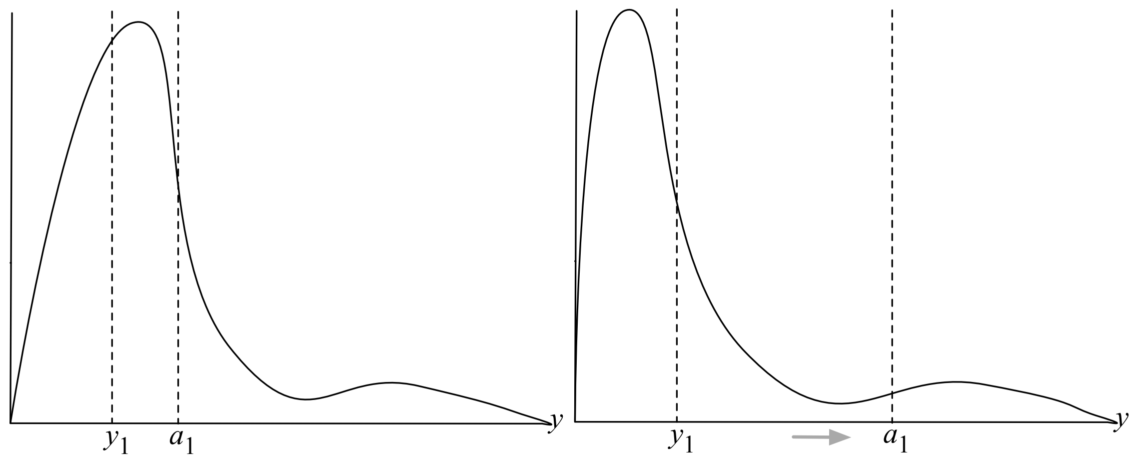
That establishes part (i). To understand parts (ii) and (iii), notice that the threshold r^* is unaffected by the aspirations formation process. Because lower aspirations (in the sense of a decrease in $\Psi(y, F)$ for every (y, F)) increase $r_0(y)$ for all y, convergence to perfect equality becomes more likely. Finally, with regard to social sensitivity, we show in the Appendix that as γ increases (i.e., as aspirations become less socially sensitive), all aspirations ratios converge to 1: those below 1 rise while those above 1 decrease. Because $r^* < 1$, lowering the social sensitivity of aspirations therefore bunches more individual aspirations ratios in an interval above r^*. That reduces the proportion of frustrated individuals, thereby increasing the likelihood of convergence to perfect equality. (Observe that without Assumption 3, $r^* > 1$ and $\underline{g} < 1$ would be possible. In this case, more — and not less — social sensitivity would reduce the proportion of frustrated individuals.)

We make two more remarks to end this Section. First, it is of some interest that in the Solow setting, perfect equality cannot be a steady state while a bimodal distribution can. Here, perfect equality can be sustained with a constant rate of exponential growth. On the other, and again in contrast to the Solow setting, an unequal distribution with a *constant* degree of relative inequality cannot persist with growth: that inequality will need to widen over time, in line with recent observations made by Piketty (2014) and his coauthors.

Second, perfect equality exhibits the highest rate of aggregate growth compared to the *asymptotic growth rate* of any other configuration. This latter rate is some convex combination of a growth rate of $\underline{g} < g(1)$ for frustrated individuals, and an asymptotic growth rate that is *at best* $g(1)$ for the satisfied individuals. (It should be noted, however, that an unequal society is *temporarily* capable of growing faster. For instance, in Case 2, the overall growth rate is a combination of rates for various aspiration ratios $g(r)$, where r begins below 1 and ends above 1. Because $g(r)$ is decreasing, it is easy to construct an example in which this combination exceeds $g(1)$.)

4.4. **Socially Sensitive Aspirations and Proposition 7.** At one level, the assumption of social sensitivity appears innocuous, only stating that aspirations rise (for given individual income) when all incomes in the society rise. But in the presence of the scale-free assumption on aspirations (which is truly innocuous), it does give rise to a tight restriction: as we move up the income ladder along some given income distribution, *individual aspirations cannot climb faster than income*: the ratio of aspirations to income must strictly fall. This is formalized as Lemma 1 above, and it is this property that implies the bimodal clustering in Proposition 6, and drives the proof of Proposition 7. But one might reasonably want to include the opposite possibility: that at least along a range of incomes in the cross-section of a distribution, aspirations rise faster than incomes do.

Suppose, for instance, that aspirations are given by the conditional mean of income above one's own income; i.e., $\Psi(y, F) = \mathbb{E}_F(x|x \geq y)$, and consider the situation depicted in Figure 8. In the first panel, there is a large mass of individuals just to the right of y_1, and so aspirations a_1 are close to y. Now move to the second panel by *lowering* all incomes by some constant factor, so that the mass of individuals just above y_1 now effectively disappear from the cognitive window of anyone located at y_1. It is entirely possible that a_1 goes *up*, but then, social sensitivity is violated.



(A) Initial Distribution

(B) Lowered Incomes

FIGURE 8. POSSIBLE FAILURE OF SOCIAL SENSITIVITY.

This example can also be "reversed" by increasing individual income y_1 keeping the overall distribution fixed. Note that aspirations will rise faster than income as y_1 crosses the large mode of the ambient distribution.

It is unclear how the dropping of social sensitivity affects the clustering proposition in Proposition 6. We conjecture that it would continue to hold generically, perhaps with a larger (but finite) number of clusters. However, we have good reason to believe that the bimodal limit described in Proposition 7 survives under a weaker assumption. Say that aspirations are *upper sensitive* if for any income y, $\Psi(y, F') > \Psi(F)$ whenever (a) F' weakly first-order stochastically dominates F: $F'(z) \leq F(z)$ for all z, (b) F' *strictly* first-order stochastically dominates F on the subdomain above y: $F'(z) < F(z)$ for all $z > y$ as long as $0 < F(z) < 1$, and (c) there are no crossings of y "from below": $F'(y) = F(y)$.

Upper sensitivity stays clear of the income crossings — emphasized in the discussion above — which serve to eliminate certain incomes from a person's "aspiration window" (in the discussion above, individuals with lower incomes lie outside that window). That is, upper sensitivity places no restriction on what happens in such cases, stating only that an individual's aspiration rises whenever all who are richer than him become yet richer, no one becomes poorer, and no poorer person becomes richer than the individual in question.

Assume upper sensitivity, and consider a version of the model as the time period becomes tiny. The following observation must hold: *once* an individual is frustrated, this state is permanent. The reason is that a frustrated individual exhibits the lowest growth rate of all observed growth rates. Moreover, in the continuous time limit, there are no crossings from below, while all incomes grow at least as fast and incomes above that individual's income grow strictly faster. By the assumption that aspirations are scale-free and upper sensitive, her aspirations must continue to grow at least as fast as her income, and her frustration must continue. This yields a monotonicity property for the fraction of frustrated individuals: that fraction cannot decline over time. Therefore it must converge. That universal frustration cannot occur in the limit can be shown by a simple extension of the argument used in the proof of Proposition 7.

Among the frustrated, there is no level convergence, but all individuals grow at the same factor $\underline{g}$. Invoking Proposition 3, we see that the satisfied exhibit a strictly higher rate of growth, thereby generating ever-widening inequality. It is now unclear that convergence occurs among the satisfied; with periodic aspirational spikes, there may well be cycles of faster and slower growth along a single dynastic path. However, applying Proposition 3 again, minimum incomes among the satisfied grow the fastest, and maximal incomes the slowest. That generates a tendency for level convergence. While it is technically possible for crossovers of income to occur in discrete time, this will not happen in the continuous time limit, and convergence of incomes (normalized by the satisfaction growth rate) must occur in the long run.

4.5. **An Empirical Exercise.** We end with an extremely tentative empirical exercise. It is tentative not because we hesitate to confront the model with the data. Rather, in a full-blown analysis we would need to be far more sensitive to actual policy and regime changes, and control for such changes before applying the details of our model.

With those qualifications, and bearing in mind that the exercise to follow is only for illustrative purposes, we ask how much our model of aspirations can capture of the *actual* variation in growth across observed income distributions. We report on two such cases.

4.5.1. *Growth Incidence Curves By Percentile for 43 Countries.* We employ a dataset from the World Bank with 55 growth incidence curves (snapshots of growth rates for every percentile in the income distribution) for 43 distinct countries.[23]

Consider the constant-elasticity growth model with $\sigma = 0.6$ and $\delta = 0.8$. In the absence of aspirations or with aspirations that are only based on one's own income, the constant-elasticity growth model would predict balanced growth across the income distribution. All income percentiles would grow at the same rate. By adjusting the return to capital ρ to match the *actual* aggregate annual rate growth observed in the data we obtain a growth incidence curve for the benchmark model without socially determined aspirations. It is flat across all incomes.

Next, we assume the aspirations formation process in (17). In country i, each percentile p's aspirations is a weighted average of p's own income (with weight $\gamma_i \in [0,1]$) and a (nested) weighted sum of the various percentile incomes in the country:

$$a_p^i = \gamma_i y_p^i + (1-\gamma_i)\sum_q w_q^i y_q^i,$$

where the weights are based on an exponential function of income: $w_q^i = \left(y_q^i\right)^{\alpha_i} / \sum_k \left(y_k^i\right)^{\alpha_i}$. Any actual income distribution together with the coefficients γ_i and α_i will generate aspirations

[23] Special thanks are due to Claudio Montenegro at the Development Research Group, Poverty Unit, The World Bank.

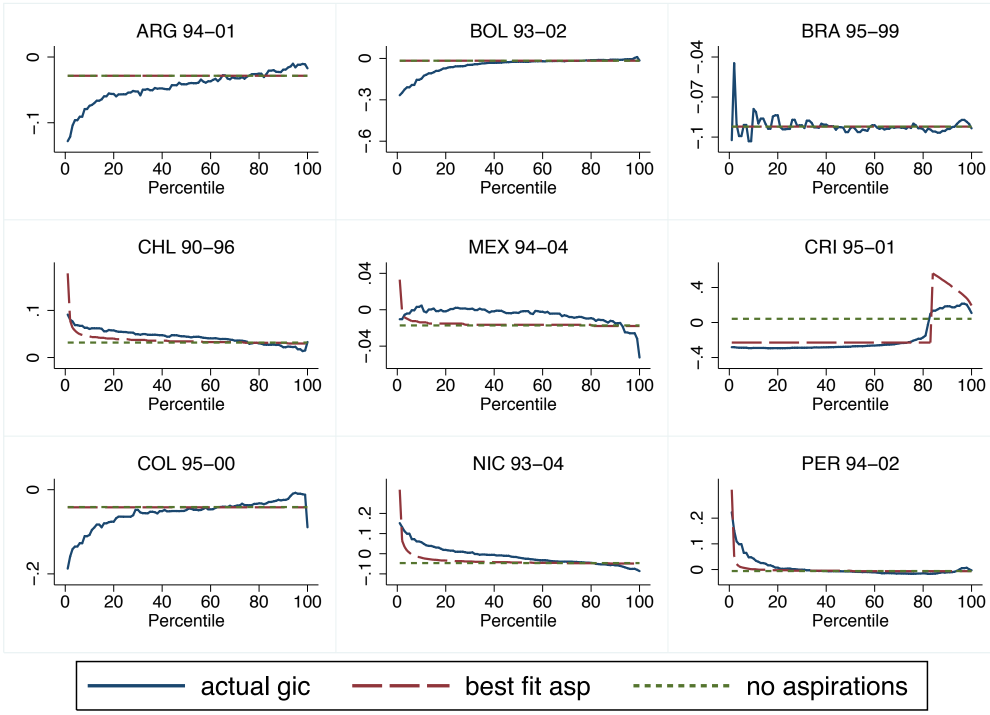


FIGURE 9. GROWTH INCIDENCE CURVES

a_p^i for each percentile p. Given these aspirations and a return to capital ρ, an individual in country i with wealth y_p^i would choose continuation wealth z_p^i to maximize

$$\left(y_p^i - \frac{z_p^i}{\rho}\right)^{1-\sigma} + \delta\left[\left(z_p^i\right)^{1-\sigma} + \pi_i\left(\max\{z_p^i - a_p^i, 0\}\right)^{1-\sigma}\right].$$

This maximization problem gives us a predicted growth incidence curve by percentile.

For each of the observations in our dataset (that is, an initial percentile distribution and a growth rate for each percentile), we consider a range of possible values for $\gamma_i \in [0,1)$, $\alpha_i \in [-8,8]$ and $\pi_i \in [0,3]$. For each combination of these parameters, we find the return to capital ρ_i that generates in our model the *actual* observed annual rate growth for country i.[24] Then, we select among all possible combinations of parameters, the one that minimizes the squared distance between the actual and predicted growth incidence curve (including the benchmark model as one of these combinations). That yields a "best fit" growth incidence curve, one for each country.

Although the limitations of this exercise are obvious, here are a few observations.

For approximately half the observed growth incidence curves, the benchmark model does a good job and our best-fit model puts no weight on aspirations ($\pi_i = 0$). For the other half, aspirations

[24]We should note that we have a degree of freedom in ρ that makes it impossible (in this model) to distinguish between a world without aspirations and aspirations that depend only on one's own income.

matter ($\pi_i > 0$) and, among these, the median weight placed by an individual her own percentile income is 0.63.

To be sure, our model is not a perfect match. Nevertheless, it captures 81% of the observed variation in growth within each distribution.[25] Figure 9 illustrates the "best fit" growth incidence curves, together with the actual growth incidence curve and the benchmark ones, for the main nine Latin American countries for which we have observations in the nineties.[26] Among these, a model without aspirations provides the best fit for 4 of them, a model with "common aspirations" ($\gamma_i = 0$) works best for one country, while for the remaining countries, the aspirations formation function puts weight on both one's own percentile and a common element. Overall our exercise is promising and suggests a scope for a rigorous calibration exercise using repeated growth incidence curves for one country and accounting for other factors that might affect the growth profile.

4.5.2. *Wealth in the United States.* In this application, we ask how well our model captures the evolution of the US wealth distribution over the period 1994–2007 using data from the Panel Study of Income Dynamics (PSID).

The total wealth of an individual i at time t y_{it} consists of the sum of her financial wealth f_{it} (including housing wealth) and the present value of her expected after-tax income ℓ_{it}.[27] Our measure of financial wealth comes from the 1994 and 2007 wealth supplement of the PSID and includes equity but does not include pension wealth.[28] We use the measure of disposable income constructed for the PSID by the Cross-National Equivalent File initiative (CNEF-PSID).[29] We restrict the sample to households with positive weights and with heads between 25 and 65 years of age. The unit of observation is the individual and all data are expressed in adult equivalents term using the modified OECD equivalence scale. To limit the importance of changing household sizes, we narrow the sample to households whose size did not change by more than 2 units over the period. Wealth and income data are expressed in real terms using the Consumer Price Index from the Bureau of Labor Statistic. To avoid outliers, we trim the sample of the few observations with wealth below one standard deviation of the lowest percentile and above one standard deviation of the highest percentile or with negative wealth. Our final sample consists of $7,526$ individuals.

We apply our constant elasticity growth model (with $\sigma = 0.8$) to the initial wealth distribution to predict the individual growth rates of wealth over the period. As in the previous section, we assume that aspirations are formed as in (17) by placing weight γ on her own initial wealth y and weight $(1-\gamma)$ on a weighted sum of all incomes where the weights are based on an exponential function of income.[30] We set the gross interest rate ρ to 3.59%, which is the geometric average

[25] This is the R^2 of a regression of the actual percentile growth on predicted percentile growth with country-year fixed effects.

[26] We will provide the graphs for all countries on request.

[27] Wealth calculations are made as follows: $y_{i1994} = f_{i1994} + (\sum_{s=0}^{12} \frac{1}{\rho^s})\ell_{i1994} + \frac{1}{\rho^{12}(\rho-1)}\ell_{i2007}$ and $y_{i2007} = f_{i2007} + \frac{\rho}{\rho-1}\ell_{i2007}$, where ρ is the gross interest rate.

[28] Earlier rounds of the PSID data do not have information on pension wealth.

[29] cnef.ehe.osu.edu/data/cnef-data-files/

[30] That is $a_i = \gamma y_{i1994} + (1-\gamma)\sum_j w^j y_{j1994}$ where $w^j = (y_{j1994})^\alpha / \sum_k (y_{k1994})^\alpha$.

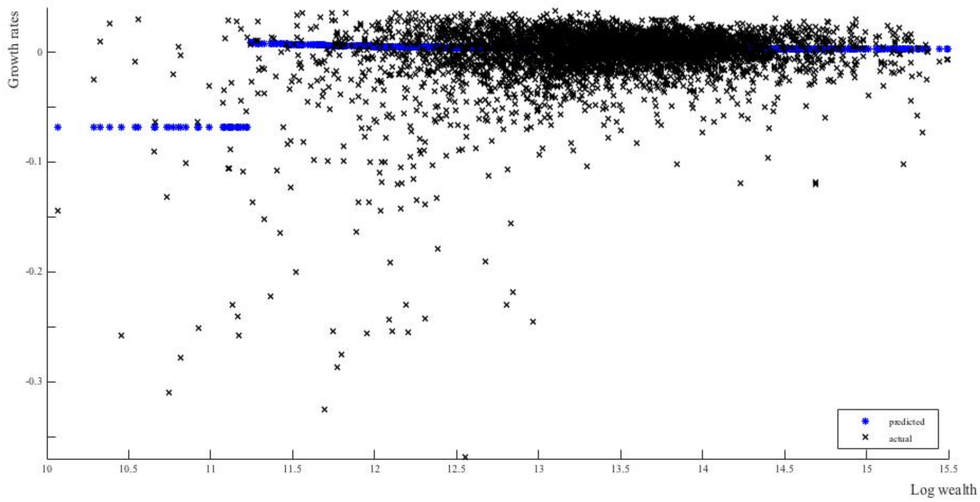


FIGURE 10. US WEALTH GROWTH RATE 1994–2007

of the annual returns on investments on 10-year Treasury bonds over the period.[31] For a given set of parameters (δ, π, γ, and α) and the initial wealth distribution, our model predicts growth rates for the wealth of each individuals. We search for the set of parameters that minimize the absolute distance between the predicted and actual annual growth rates in individual wealth.

Figure 10 shows the resulting growth incidence curve as well as the actual growth rates as a function of log initial wealth. If aspirations did not matter; i.e., $\pi = 0$, or if everyone had aspirations equal to their level of income $\gamma = 1$, the model would predict the same growth rate for each quintile. In contrast, we find that individuals put significant weight on aspirations: $\pi = 0.40$. Within the aspirations term, one's own wealth carries substantial weight — $\gamma = 0.95$ — while a weight of $\alpha = -2.2$ for the incomes in the common aspirations component sets that component equal to $\$151,346$ in 1994. Finally, the model estimates a discount factor δ of 0.92 per year. In contrast to the rest of the population, the poorest 3% of the population are frustrated and see their wealth decline over the period.

In this exercise, we have assumed that all individuals have access to the same return to capital. This is clearly not realistic as one expect the richest segments of the population to have access to higher rates of return. While we do not attempt to fit such a model, we do not expect the results to change much, because the benchmark version of such a specification (with no aspirations) would predict increasing growth rates among the richest half of the population and therefore not fit well the actual distribution. Thus, although the limitations of this exercise are obvious, it suggests that aspirations could play a role in explaining the variation in US growth rates over this period.

[31]Using the arithmetic average of 3.89% hardly affects the results. The annual returns can be found at http://people.stern.nyu.edu/adamodar/New_Home_Page/datafile/histretSP.html

5. CONCLUSION

This paper builds a theory of aspirations formation. The theory emphasizes the social foundations of individual aspirations, and relates those aspirations in turn to investment and growth. Following a familiar lead from behavioral economics (see, e.g., Kahneman and Tversky (1979), Karandikar et al. (1998), and Kőszegi and Rabin (2006)), we define utilities around "reference points," and interpret these reference points as *aspirations*. However, we depart from existing literature in two fundamental ways. First, we focus on the social determination of aspirations, in contrast to the private experiences of the individual herself, or some self-fulfilling belief about what she expects. We argue that aspirations are as likely to depend on the experience and lifestyle of others. In our framework, both private and social considerations enter into the determination of aspirations.

Second, aspirations determine an individual's incentives to invest and bequeath. Such behavior can be aggregated across individuals to derive the society-wide distribution of income, thus closing the model. This equilibrium interplay between the individual and the social is a main theme of the paper.

A central feature of our theory is that aspirations can serve both to incentivize and to frustrate. We show that aspirations that are above — but not too far — from current incomes can encourage high investment, while aspirations that are too high may discourage it. Hence, rising aspirations not only decrease individual utilities but can also lack instrumental value. This insight has implications for growth rates across a cross-section of aspirations for a given starting income, as well as for growth rates across a cross-section of incomes, for a given level of aspirations.

A study of society-wide equilibrium leads to additional insights. In the Solow setting with capital stocks in some compact set, steady state distributions must exhibit inequality. When aspirations are socially sensitive, in that they respond positively to an increase in society-wide incomes, the distributions must also be bimodal.

In the constant-elasticity growth model, sustained growth is possible. When aspirations are socially sensitive, there are only two outcomes possible: either convergence to an equal distribution (with growth) or perennial relative divergence with two components, so that ever-expanding inequality is the result. The resulting growth rate of per-capita income is strictly lower than the rate achieved under perfect equality.

The goal of this paper has been to model aspirations as a socially determined set of reference points. Our model has the advantage of being tractable and has allowed us to explore the relationship between aspirations and inequality. We believe that the simplicity of the framework is also conducive to several extensions. We mention a few of interest to us. One direction is a theory of *group-based* aspirations, in which different social or ethnic groups draw their reference points in different ways from society. For instance, Munshi and Myaux (2006) argue that fertility behavior among particular religious groups in Bangladesh influence fertility norms for couples in the same religious groups, but not across groups. This sort of study also suggests a second extension, in which aspirations themselves are multidimensional: not some narrow scalar notion such as wealth, as explored here, but an entire complex that might include education, fertility or social achievement. Developing the model along these lines would tie these ideas in with the notion of capabilities developed by Sen (1985), except that such "capabilities" would, in part, appear as a relativistic construct, inspired by the achievements of others. A third extension, also influenced in part by the notion of group-based aspirations, would link aspirations to frustration and subsequent violence. Presumably these models would extend the simple allocative exercise in this paper to a three-way allocation across consumption, productive investment, and resources spent in social conflict. This sort of theory would tie into recent empirical analyses of uneven growth and conflict, such as Dube and Vargas (2013) and Mitra and Ray (2014). Finally, if one is willing to take these models a bit more literally, it is possible to use growth incidence curves (say, by decile or percentile), along with some controls to account for major policy or regime shifts, to actually estimate the aspirations-formation process for different societies.

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APPENDIX: PROOFS

Proof of Proposition 2. It follows from the continuity of $w_0(z)$ for $z \geq 0$ and the fact that $w_0'(0) = \infty$ that there is a range of aspirations starting at 0 so that aspirations are satisfied. Similarly, $u'(0) = \infty$ implies that for aspirations levels high enough aspirations must be frustrated. Moreover, the fact that $w_1(z - a)$ is decreasing in a implies that if an individual with income y has frustrated aspirations at a, he must have frustrated aspirations at any $a' > a$. In particular, there is a unique level of aspiration a^* so that aspirations are satisfied for all $a < a^*$ and frustrated for all $a > a^*$.

The first order condition (4) makes clear that once $a > a^*$ a further increase in aspirations has no effect on the marginal utility of wealth and so fails to encourage wealth accumulation.

On the other hand, we see from the first order condition (5) that an increase in aspirations incentivizes growth as long as aspirations remain in the "satisfaction zone." ■

Proof of Lemma 1. Consider two incomes y_1 and y_2 in the support of F, with $y_2 = \lambda y_1$, where $\lambda > 1$. If aspirations are socially sensitive, $\Psi(y_2, F) < \Psi(y_2, \lambda F)$, where λF is obtained from F by scaling all incomes up by λ. It follows that

$$r(y_2, F) = \frac{y_2}{\Psi(y_2, F)} > \frac{y_2}{\Psi(y_2, \lambda F)} = \frac{\lambda y_1}{\Psi(\lambda y_1, \lambda F)} = \frac{y_1}{\Psi(y_1, F)} = r(y_1, F),$$

where the equality $\frac{\lambda y_1}{\Psi(\lambda y_1, \lambda F)} = \frac{y_1}{\Psi(y_1, F)}$ follows from the assumption that aspirations are scale-free. ■

Proof of Proposition 3. If an individual with $r \equiv y/a$ choses a growth rate $g = g(r) > 1/r$, it implies that

$$\left(1 - \frac{g}{\rho}\right)^{1-\sigma} + \delta \left[g^{1-\sigma} + \pi \left(g - \frac{1}{r}\right)^{1-\sigma}\right] \geq \left(1 - \frac{\underline{g}}{\rho}\right)^{1-\sigma} + \delta \underline{g}^{1-\sigma}.$$

Since the left-hand side is strictly increasing in r, it implies that any other individual with $r' = y'/a > a$ has satisfied aspirations as well and chooses the growth rate $g(r')$ that solves (10). Hence, there is a unique threshold r^* such that for all wealth-aspiration pairs (y, a) with $r \equiv y/a < r^*$, continuation wealth grows by the factor $\underline{g}$, and for all (y, a) with $y/a > r^*$, continuation wealth grows by the factor $g(y/a)$.

In addition, $r^* < 1$ follows from restriction 3. Indeed, 3 implies that $\underline{g} > 1$. Hence, if it was the case that $r^* \geq 1$, there would exist $r = r^* - \epsilon$ for a sufficiently small $\epsilon > 0$ so that $r\underline{g} \geq 1$ with contradicts the premise that an individual at $r(¡r^*)$ is frustrated. Hence, $r^* < 1$.

By comparing (10) and (11), it is easy to see that $g(r) > \underline{g}$. Moreover, an inspection of (10) shows that $g(r)$ strictly declines as r rises. At the same time, if we pass to the limit as $y \to \infty$

(so that $r \to \infty$), $g(r)$ is bounded away from $\underline{g}$ (set $[1/r] = 0$ in (10) and compare the condition with (11)). It follows that once $y > r^* a$, $g(y/a)$ declines in y but is always strictly larger and bounded away from $\underline{g}$ on $y \in (y(a), \infty)$. ■

Proof of Proposition 5.

If F^* is concentrated on a single point y^*; then, by range-boundedness, steady state aspirations must be given by $a = y^*$. But, we have already noted that there are at most two possible optimal choices at any level of wealth y and aspiration a. However, neither of these choices can be equal to a itself. For $z = a$ to happen, we must have

$$w_0'(a) \geq u'(y - k(a))/f'(k(a)),$$

while at the same time,

$$w_0'(a) + w_1'(0) \leq u'(y - k(a))/f'(k(a)).$$

Given that $w_1'(0) > 0$, both these inequalities cannot simultaneously hold. ■

Proof of Proposition 6. First, we show that

Lemma 2. *If aspirations are scale-free and socially sensitive, $y - \Psi(y, F)$ is strictly increasing in y.*

Proof. Consider two incomes y_1 and y_2 in the support of F, with $y_2 = \lambda y_1$, where $\lambda > 1$. Scale-free aspirations means that $\Psi(y_1, F) = \Psi(y_2, \lambda F)$ while social sensitivity implies that $\Psi(y_2, F) < \Psi(y_2, \lambda F)$. Hence,

$$y_1 - \Psi(y_1, F) < \lambda y_1 - \lambda\Psi(y_1, F) = y_2 - \Psi(y_2, \lambda F) < y_2 - \Psi(y_2, F).$$

■

We can now proceed with the main proof.

Fix a steady state distribution F. Consider any two distinct incomes in the support of F. We claim that one of them must have unsatisfied aspirations, and is the *unique* solution to

$$-\frac{u'(y - k(y))}{f'(k(y))} + w_0'(y) = 0. \tag{18}$$

while the other must have satisfied aspirations, and is the *unique* solution to

$$-\frac{u'(y - k(y))}{f'(k(y))} + w_0'(y) + w_1'(y - \Psi(y, F)) = 0. \tag{19}$$

Because the solutions to (18) and (19) are unique (as we shall show), the proof is complete.

The fact that (18) admits a unique solution follows immediately from the assumption that the benchmark stationary state is unique (Condition D). For any benchmark stationary state is described by (18). Next, suppose on the contrary that (19) admits two solutions y_1 and y_2, with $y_1 < y_2$. By Lemma 2, we have $y_2 - \Psi(y_2, F) > y_1 - \Psi(y_1, F)$, and using the strict concavity of w_1, it follows that

$$w_1'(y_1 - \Psi(y_1, F)) > w_1'(y_2 - \Psi(y_2, F)).$$

Because (19) holds for both y_1 and y_2, we must conclude that

$$-\frac{u'(y_2 - k(y_2))}{f'(k(y_2))} + w_0'(y_2) > -\frac{u'(y_1 - k(y_1))}{f'(k(y_1))} + w_0'(y_1),$$

which contradicts Condition D. ■

Proof of Proposition 7. Our proof will rely on the following observations:

Lemma 3. *Generate F' from F by multiplying every wealth y in the support of F by a growth factor $g(y)$, then for each y and each $\alpha > 0$, $\Psi(\alpha y, F')$ differs from $\Psi(y, F)$ by a factor that lies in* $[\min\{\alpha, \inf_x g(x)\}, \max\{\alpha, \sup_x g(x)\}]$.

Proof. Let $a = \Psi(y, F)$ and $a' = \Psi(\alpha y, F')$. To prove that $a'/a \geq \min\{\alpha, \inf_x g(x)\}$, consider an intermediate step in which we move to a distribution F'' by multiplying all incomes in F by $\lambda = \min\{\alpha, \inf_x g(x)\}$. Because aspirations are scale-free, $a'' = \Psi(\lambda y, F'')$ must *equal* λa. But now observe that $(\alpha y, F')$ differs from $(\lambda y, F'')$ only by an (possible) additional increase of incomes. Because aspirations are nondecreasing in (y, F), we have $a' \geq a''$. It follows that $a' \geq \min\{\alpha, \inf_x g(x)\}a$. The proof that $a' \leq \max\{\alpha, \sup_x g(x)\}a$ employs a very similar argument, and is omitted. ■

Define aspirations ratios for each income and at each date by

$$r_t(y) \equiv y/\Psi(y, F_t) \text{ for every } y \in \text{Supp } F_t.$$

By Lemma 1, we know that for every date t, $r_t(y)$ is strictly increasing in y.

Lemma 4. *Suppose that two incomes y_0 and y_0', with $y_0 \leq y_0'$, both have satisfied aspirations under the distribution F_0: $r_0(y_0) \geq r^*$ and $r_0(y_0') \geq r^*$. Then the lower income will grow weakly faster, but nevertheless $y_1 \leq y_1'$.*

Proof. Consider y_0 and y_0' as described in the statement of the Lemma. By Lemma 1, $r_0(y_0) \leq r_0(y_0')$, so given that aspirations are satisfied for both incomes, Proposition 3 tells us that growth rates at y_0 are weakly higher. At the same time, given that aspirations are nondecreasing in own income, the aspiration of the person at income y_0' is higher than at y_0. By invoking Proposition 2 (continuation wealth nondecreasing in aspirations as long as aspirations are satisfied) and a standard single-crossing property (continuation wealth is nondecreasing in initial wealth for fixed aspirations), we must conclude that the continuation wealth y_1' from y_0' exceeds continuation wealth y_1 starting from y_0. ■

Lemma 5. *Recall the value r^* defined in Proposition 3. If $r_0(y) < r^*$, then along the equilibrium path starting from y, aspirations are frustrated at every date. If $r_0(y) \geq r^*$, then along the equilibrium path starting from y, aspirations are satisfied at every date.*

Proof. Suppose that $r_0(y) < r^*$, then aspirations are certainly frustrated at date 0 and the growth factor at y is $\underline{g}$. By Proposition 3, this is the lowest of all growth rates in equilibrium. By Lemma 3, aspirations at income y must grow by some factor that is at least $\underline{g}$. It follows that $r_1(\underline{g}y) \leq r_0(y) < r^*$, and aspirations are frustrated again. This argument can now be extended to every period.

For the second part, define $\underline{r}_0$ to be the infimum of aspirations ratios that are no less than r^* under the distribution F_0. By Proposition 3, an individual located at $\underline{r}_0$ will exhibit the highest growth factor of incomes; call it $\bar{g}$.[32] By Lemma 3, aspirations at the income $\underline{y}_0$ corresponding to this aspirations ratio $\underline{r}_0$ must grow by some factor that is *at most* $\bar{g}$. Therefore, if we let $\underline{r}_1$ stand for the subsequent aspirations ratio (starting from $\underline{r}_0$) at date 1, we have

$$\underline{r}_1 \geq \underline{r}_0 \geq r^*. \tag{20}$$

Now consider any y_0 with $r_0(y) \geq r^*$. By Lemma 1, we have $y_0 \geq \underline{y}_0$, where $\underline{y}_0$ is the starting wealth associated with the ratio $\underline{r}_0$. By Lemma 4, $y_1 \geq \underline{y}_1$. Again applying Lemma 1, we have $r_1(y_1) \geq \underline{r}_1$. Combining this observation with (20), we must conclude that

$$r_1(y_1) \geq \underline{r}_1 \geq \underline{r}_0 \geq r^*.$$

It follows that aspirations are also satisfied in the next period for the person with initial income y_0. This argument can now be extended to every period. ■

Returning to the main proof, we consider three cases:

Case 0: $\sup_y r_0(y) \leq r^*$. In this case, *every* individual has frustrated aspirations under F_0. But now we claim that $\underline{g} < 1$. To see this, note that because aspirations are range-bound, $r_0(y) \geq 1$ for some income y in the support of F_0. It follows that if $\underline{g} \geq 1$, then the optimal solution for y must lie above $a = \Psi(y, F_0)$. That contradicts our premise that all individuals have $r_0(y) \leq r^*$.

Case 1: $\inf_y r_0(y) \geq r^*$. In this case, by Lemma 5, aspirations will remain satisfied for every individual forever. So *every* individual, starting from their initial values of y_0, chooses the high accumulation path with successive growth factors $\{g(r_t(y_t))\}$, as given by (10).

By Lemma 4, the rank of incomes is preserved at every date, and by Lemma 1 $r_t(y_t)$ is strictly increasing in y_t. As seen in Proposition 3, $g(r)$ is strictly decreasing, so that growth rates are strictly decreasing in income. Let m_t be be the ratio of infimum to supremum incomes at date t. Because lower incomes grow strictly faster, m_t is strictly increasing and bounded above by 1, so it must converge. But in that limit the lowest income must be growing at the same rate as the highest income. Because $g(r)$ is strictly decreasing, that can only happen if the limit equals 1. In particular, all aspiration ratios under the sequence F_t must converge to 1. It follows that growth rate converges to $g(1)$. That $g(1) > 1$ follows from inspecting (10) with r set equal to 1.

Because we eliminate the case in which all individuals have frustrated aspirations under F_0, the only remaining possibility is:

Case 2. : $\inf_y r_0(y) < r^* < \sup_y r_0(y)$. By Lemma 5, the growth factor of all individuals with $r_0(y) < r^*$ must be $\underline{g}$ in all periods. By contrast, every individual with $r_0(y) \geq r^*$ must grow at a strictly higher rate than $\underline{g}$; indeed, bounded away from $\underline{g}$ (Proposition 3). That proves that inequality between the frustrated and the satisfied must grow unboundedly high with time.

As for those individuals who are satisfied at date 0, we follow an argument similar to that for Case 1. Each such individual chooses the high accumulation path with successive growth factors $\{g(r_t(y_t))\}$. By Lemma 4, the rank of incomes is preserved at every date, and by Proposition 3

[32] We invoke the convention that the upper choice is made at r^*, in case $\underline{r}_0 = r^*$.

and Lemma 1, lower initial incomes grow strictly faster. Define m_t as in Case 1, but with the infimum taken only over the set of satisfied individuals. As before, m_t converges to some finite limit. In that limit the lowest income must be growing at the same rate as the highest income (which is well-defined, as F_0 has compact support), so that the limit of m_t can only be 1.

The above argument also shows that the limit rate of growth of all satisfied individuals is the same. Denote this limit growth factor by $\bar{g}$. By Proposition 3, we know that $\bar{g} > \underline{g}$. By range-boundedness, the aspirations ratio of the supremum income is no smaller than 1, so $\bar{g} = g(\bar{r})$ for some $\bar{r} \geq 1$. By Proposition 3 again, $\bar{g} \leq g(1)$. This proves the very last claim of the Proposition. ■

Proof of Observation 1, Part iii. We prove the following Lemma:

Lemma 6. *$r_t(y)$ is strictly increasing (decreasing) in γ if $r_t(y) < (>) 1$, and bounded above (below) by 1.*

Proof. Using (17), aspirations ratios at time t are

$$r_t(y) \equiv \frac{1}{\gamma + (1-\gamma)\psi(F_t)/y} \text{ for every } y \in \text{Supp } F_t.$$

The effect of γ is given by $dr_t(y)/d\gamma = -r_t(y)^2(1 - \psi(F_t)/y)$. Hence, an increase in γ raises (lowers) $r_t(y)$ if $\psi(F_t) > (<) y$. Since the latter inequality depends on how r_t compares to 1,

$$\frac{dr_t(y)}{d\gamma} > (=, <) 0 \text{ for } r_t(y) < (=, >) 1,$$

while $r_t(y)$ is correspondingly bounded above (below) by 1. ■

Proposition 3 shows that whether an individual with income y is frustrated or satisfied at time 0 (and therefore at any future date as seen in Proposition 7) depends on whether $r_0(y)$ is lower or higher than the threshold r^* identified in Proposition 3. Moreover, this threshold r^* is smaller than 1, and is not affected by γ results in a greater measure of aspirations ratios exceeding r^* at date 0. For any given initial distribution F_0, that reduces the proportion of frustrated individuals at date 0, and so makes convergence to perfect equality more likely. ■